Gimballess Attitude Control Systems for Spacecraft Using a Spherical Rotor

Ayaka Mino and Kenji Uchiyama
Nihon University, Chiba, Japan
e-mail: csay17037@g.nihon-u.ac.jp, uchiyama@aero.cst.nihon-u.ac.jp

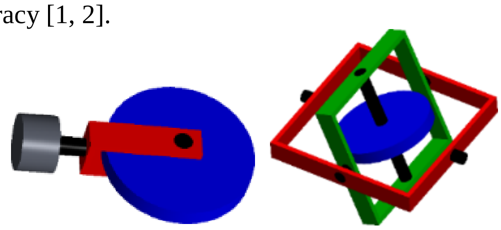
***Abstract*—A new control moment gyro (CMG) using a gimballess design is developed for attitude control of a spacecraft. Although the benefits of conventional CMG systems include cleanliness, the replenishable power source, and generation of a high torque to change the direction of a spacecraft, it is necessary to design a complex control system to avoid impassable and escapable singular states that are caused by gimbals. The proposed CMG system using a spherical rotor does not require gimbals for three-axis attitude control. The spherical rotor, which is driven by three pairs of omni wheels attached at a shaft of a motor, consists of a spherical shell, a flywheel, a battery, and an Inertial measurement Unit (IMU) to measure the angular velocity of the rotor. Singularity analysis of the proposed CMG is described, and a preliminary experiment is performed to verify the validity of the CMG.**

***Index Terms*—Spherical CMG, gimballess design, attitude control device, momentum exchange, omni wheel.**

I. Introduction

There are various control devices that provide an effective means of reorienting spacecraft: reaction wheels (RWs), control moment gyros (CMGs), and magnetic torquers (MTQs). Most spacecraft utilize either RWs or CMGs, which are momentum exchange devices. The use of RWs as an actuator is a feasible and relatively simple method for controlling the attitude of a spacecraft. RWs are useful in cases where precise attitude control is required for a long period of time and where electrical energy is to be generated by means of solar cells. RWs are usually implemented as electric motors mounted along at least three directions along the axes of a body reference frame. It should be noted that external forces on a spacecraft would require a gradual buildup of rotational speed of a flywheel to maintain the spacecraft in the desired orientation. On the other hand, CMG systems are well known as an effective torque generator for attitude control for large space structures such as space stations. A gimbal arrangement of CMG allows the variation of a spin axis in a body reference frame. In particular, the rate of gimbal spin can produce an output torque orthogonal to both the gimbal and the spin axes, which is much greater than the gimbal axis torque.

The gimbal structure of a CMG system is divided into two types depending on the degrees of freedom; a single-gimbal CMG (SGCMG) and a double-gimbal CMG (DGCMG), as shown in Figs. 1(a) and 1(b), respectively. Reorientation of a spin axis is restricted in a plane that is perpendicular to the gimbal axis, even though SGCMG has the advantage of torque amplification property. In the case of DGCMG, the system can generate an output torque to an arbitrary direction in a plane that is orthogonal to the direction of a flywheel by changing its angular momentum around two gimbal directions. A DGCMG system generally has size, weight, and power advantages over SGCMG. Moreover, when using gimbals to change the angular velocity of a flywheel, a singularity is encountered when some directions of a CMG system are not capable of generating torque. Therefore, the CMG system should be provided with complex steering laws for practical operation, despite its favorable performance. In order to cope with this problem, the system is designed to have redundancy or singularity avoiding/passing algorithms that are used at the expense of control accuracy [1, 2].



(a) Single gimbal (b) Double gimbal

Figure 1. Gimbal structure for CMG.

In this paper, we propose a novel CMG system that has no singularities in terms of the directions of the generated torque because it employs a gimballess design. The developed CMG system consists of a spherical shell containing a flywheel, a battery, an IMU that measures the rotational speed of the sphere, and three pairs of omni wheels attached at a shaft of a motor to rotate the sphere. The characteristics of the proposed system are the absence of the singularity and the control of the direction of a spacecraft with a simple controller.

II. Singularity Analysis

At least three SGCMGs are required to perform three axes of attitude control because the torque generated by the SGCMG is around only one axis. In this paper, we analyze the singularity of a skew pyramid-type CMG cluster that is commonly used for avoidance and fault tolerance, shown in Fig. 2 [3]. In this figure, $\mathbf{g}_i$ and

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δ_i $(i = 1, \ldots, 4)$ denote the gimbal vector and angle of the i th CMG, respectively. The total angular momentum $\mathbf{H}$ is expressed as

$$\mathbf{H} = \sum_{i=1}^{4} \mathbf{h}_i(\delta_i)\,, \tag{1}$$

$$\mathbf{H} = \begin{bmatrix} -c\beta s\delta_1 & -c\delta_2 & c\beta s\delta_3 & c\delta_4 \\ c\delta_1 & -c\beta s\delta_2 & -c\delta_3 & c\beta s\delta_4 \\ s\beta s\delta_1 & s\beta s\delta_2 & s\beta s\delta_3 & s\beta s\delta_4 \end{bmatrix}. \tag{2}$$

Here, $\mathbf{h}_i$ $(i = 1, \ldots, 4)$ is the angular momentum vector of the CMG at the ith position of the spacecraft as shown in Fig. 2, $\mathrm{s}(\cdot) = \sin(\cdot)$, and $\mathrm{c}(\cdot) = \cos(\cdot)$.

$$\mathbf{A} = \begin{bmatrix} -c\beta c\delta_1 & s\delta_2 & c\beta c\delta_3 & -s\delta_4 \\ -s\delta_1 & -c\beta c\delta_2 & s\delta_3 & c\beta c\delta_4 \\ s\beta c\delta_1 & s\beta c\delta_2 & s\beta c\delta_3 & s\beta c\delta_4 \end{bmatrix} \tag{3}$$

where $\mathbf{A}$ is a Jacobian matrix. The system has a singular direction if $\mathbf{A}$ is an irregular matrix [4].

Figure 3 shows the result of singularity analysis for a skew angle $\beta = 54.73$ deg. A passable surface and an impassable surface are indicated by the purple points and the yellow points in the figure, respectively. It can be seen that a number of singularities occur in the pyramid-type CMG, whether it is passable or not.

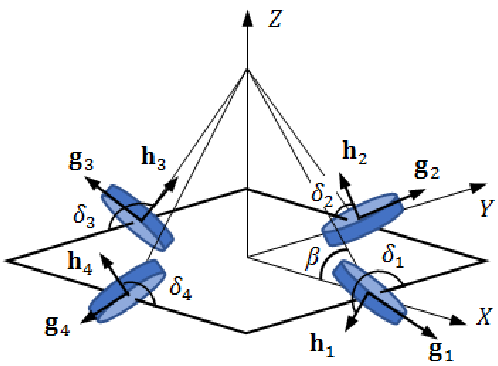


Figure 2. Configuration of the pyramid-type CMG system.

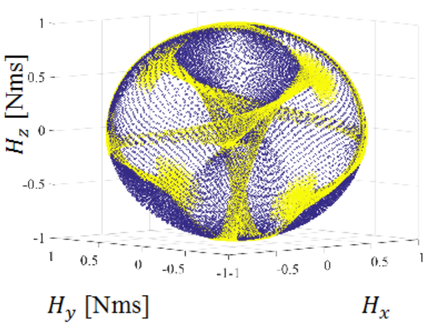


Figure 3. Singular surface in three-dimensional space.

III. Spherical CMG

A. Structure

It is necessary for the conventional CMG system to use gimbals to control the attitude of a spacecraft three-dimensionally. However, using CMGs with a gimbal mechanism, the singularities and gimbal locks are inevitable problems that are difficult to solve structurally.

Figure 4(a) shows the proposed system without the gimbal structure. Motors attached to each surface of the rectangular parallelepiped drive the spherical rotor containing the constantly spinning flywheel. The conventional SGCMG generates torque to orientate the spin axis only in a plane that is perpendicular to the gimbal axis. The spherical CMG containing the flywheel and an IMU generates torque in every direction.

B. Omni Wheel

Omni wheels [shown in Fig. 4(b)] that play a role in driving a sphere freely around three axes are applied to the system. Omni wheels have small barrel-shaped rings on the circumference of a main wheel. Those rotational directions are perpendicular to the turning direction of the main wheel. The omni wheel gives proper force to the sphere and allows the sphere to slide laterally with ease. Omni wheels are assigned on each axis of the orthogonal coordinate, as shown in Fig. 4(b).

C. Driving System

Figure 5 shows the driving principle of the proposed system. Omni wheels attached to DC motors transmit power using friction force acting on the spherical shell. The combination of the forces that are generated by each actuator revolves about the sphere around arbitrary axes. The angular velocities and rotational angles on XY, YZ, and ZX planes are defined as ω_{xy} and θ_{xy}, ω_{yz} and θ_{yz}, and ω_{zx} and θ_{zx}, respectively, as shown in Fig. 6. A pair of motors are used to revolve the sphere around an axis for high torque and fault tolerance.

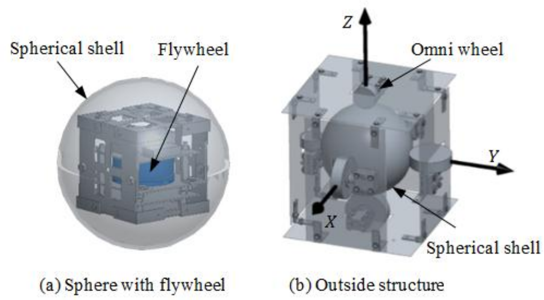


Figure 4. Spherical CMG.

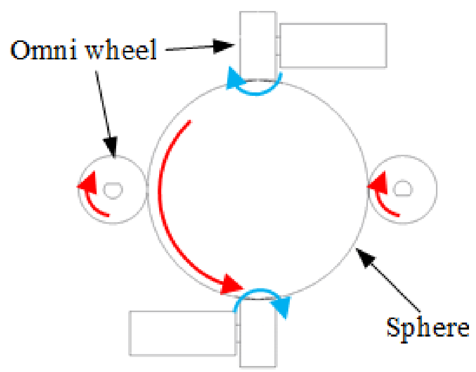


Figure 5. Driving principle of the proposed system.

IV. Gyroscopic Effect of Spherical CMG

A. Driving Principle

The rotational direction of the flywheel can be varied by revolving the spherical rotor. The gyroscopic effect generated by the spherical CMG is used to change the direction of angular momentum of a spacecraft. As mentioned in the previous section, the angular velocities that are used to calculate the gyroscopic effect are defined.

The flywheel revolves around the Y-axis and its angular velocity ω_w is shown in Fig. 7. When three pairs of omni wheels drive the sphere, the angular momentum of the flywheel $\mathbf{L}_\mathrm{w}$ and the angular velocity of the sphere $\mathbf{\Omega}_\mathrm{s}$ are expressed as follows:

$$\mathbf{L}_\mathrm{w} = \begin{bmatrix} I_w(\omega_{xy}\mathrm{c}\theta_{xy} + \omega_{zx}\mathrm{s}\theta_{zx}) \\ J_w(\omega_\mathrm{w} + \omega_{yz}\mathrm{c}\theta_{yz} + \omega_{xy}\mathrm{s}\theta_{xy}) \\ K_w(\omega_{zx}\mathrm{c}\theta_{zx} + \omega_{yz}\mathrm{s}\theta_{yz}) \end{bmatrix} \tag{4}$$

$$\mathbf{\Omega}_\mathrm{s} = \begin{bmatrix} \omega_{xy}\mathrm{c}\theta_{xy} + \omega_{zx}\mathrm{s}\theta_{zx} \\ \omega_{yz}\mathrm{c}\theta_{yz} + \omega_{xy}\mathrm{s}\theta_{xy} \\ \omega_{zx}\mathrm{c}\theta_{zx} + \omega_{yz}\mathrm{s}\theta_{yz} \end{bmatrix} \tag{5}$$

where I_w, J_w, and K_w are the moment of inertia of the flywheel around X, Y, and Z, respectively. From the above equations, the gyroscopic effect with rotation of the sphere can be calculated using the following equation:

$$\Delta\mathbf{L} = \mathbf{\Omega}_\mathrm{s} \times \mathbf{L}_\mathrm{w}. \tag{6}$$

The gyroscopic effect in each axis is given as

$$\Delta L_x = (I_w - J_w)(\omega_{zx}\mathrm{c}\theta_{zx} + \omega_{yz}\mathrm{s}\theta_{yz})(\omega_{yz}\mathrm{c}\theta_{yz} + \omega_{xy}\mathrm{s}\theta_{xy}) - J_w(\omega_{zx}\mathrm{c}\theta_{zx} + \omega_{yz}\mathrm{s}\theta_{yz}), \tag{7}$$

$$\Delta L_y = 0, \tag{8}$$

$$\Delta L_z = (J_w - I_w)(\omega_{xy}\mathrm{c}\theta_{xy} + \omega_{zx}\mathrm{s}\theta_{zx})(\omega_{yz}\mathrm{c}\theta_{yz} + \omega_{xy}\mathrm{s}\theta_{xy}) + J_w(\omega_{xy}\mathrm{c}\theta_{xy} + \omega_{zx}\mathrm{s}\theta_{zx}). \tag{9}$$

It is assumed that $I_w = K_w$. The second component of the matrix is in the same direction of the rotational axis of the flywheel (i.e., the gyroscopic effect around the axis cannot be generated). The system can generate the output torque calculated by Eqs. (7) and (9). It is, however, necessary to analyze singularities with respect to the rotational direction.

According to the results of the analysis, there exist a few singular points. The system cannot generate the output torque around the X-axis when the angular velocity around the Z-axis is zero (i.e., $\omega_{yz} = \omega_y$). Similarly, the torque around the Z-axis cannot be generated when $\omega_{xy} = \omega_y$. However, the proposed spherical CMG system can basically avoid those singularities as long as the sphere is driven by three pairs of omni wheels. It should be noted that a driving system with fault tolerance is required for generating torque around an arbitrary axis.

B. Driving the Sphere Using Two Pairs of Omni Wheels

Here, we discuss the gyroscopic effect when using two pairs of omni wheels. In this case, we do not use a pair of omni wheels whose direction agrees with the rotational direction of the flywheel. The angular momentum $\mathbf{L}_\mathrm{w}$ and the angular velocity of the sphere $\mathbf{\Omega}_\mathrm{s}$ are obtained as follows:

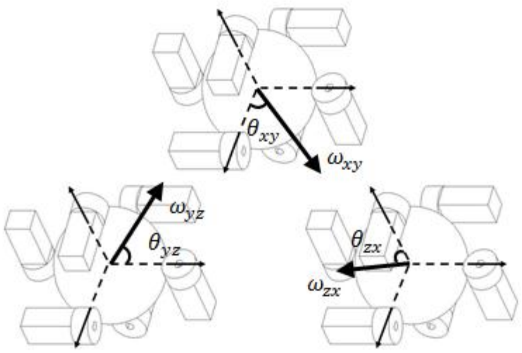


Figure 6. Angular velocity vector of a spherical CMG.

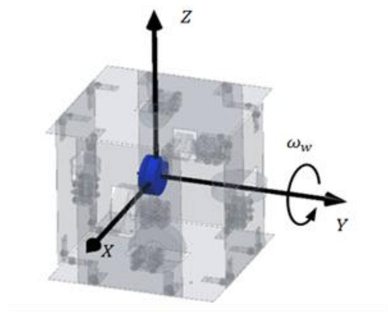


Figure 7. Angular velocity of flywheel in a spherical CMG.

$$\mathbf{L}_\mathrm{w} = \begin{bmatrix} I_w\omega_{zx}\mathrm{s}\theta_{zx} \\ J_w\omega_w \\ I_w\omega_{zx}\mathrm{c}\theta_{zx} \end{bmatrix}, \tag{10}$$

$$\mathbf{\Omega}_\mathrm{s} = \begin{bmatrix} \omega_{zx}\mathrm{s}\theta_{zx} \\ 0 \\ \omega_{zy}\mathrm{c}\theta_{zx} \end{bmatrix}. \tag{11}$$

From Eqs. (5), (9), and (10), the gyroscopic effect with rotation of the sphere is expressed as follows:

$$\Delta L_x = -J_w\omega_w\omega_{zy}\mathrm{c}\theta_{zx}, \tag{12}$$

$$\Delta L_y = 0, \tag{13}$$

$$\Delta L_z = J_w\omega_w\omega_{zy}\mathrm{s}\theta_{zx}. \tag{14}$$

It is clear that the system can generate an output torque except around the Y-axis. However, the directions of the generated torque are constrained around the X-axis or Z-axis when $\omega_{zx} = \omega_x$ or $\omega_{zx} = \omega_z$.

It is important to discuss the gyroscopic effect when the rotational direction of a pair of omni wheels coincides with one of the flywheels. The angular momentum of the flywheel $\mathbf{L}_\mathrm{w}$ and the angular velocity of the sphere $\mathbf{\Omega}_\mathrm{s}$ for driving omni wheels around the X-axis and Y-axis are expressed in the following equations:

$$\mathbf{L}_\mathrm{w} = \begin{bmatrix} I_w \omega_{xy} c\theta_{xy} \\ J_w(\omega_w + \omega_{xy} s\theta_{xy}) \\ 0 \end{bmatrix}, \tag{15}$$

$$\mathbf{\Omega}_\mathrm{s} = \begin{bmatrix} \omega_{xy} c\theta_{xy} \\ \omega_{xy} s\theta_{xy} \\ 0 \end{bmatrix}. \tag{16}$$

Thus, the gyroscopic effect with rotation of the sphere can be obtained as follows:

$$\Delta L_x = 0, \tag{17}$$

$$\Delta L_y = 0, \tag{18}$$

$$\Delta L_z = (J_w - I_w)\omega_{xy}{}^2 s\theta_{xy} c\theta_{xy} + J_w \omega_w \omega_{xy} c\theta_{xy}. \tag{19}$$

When the direction of the omni wheel agrees with the rotational axis of the flywheel, the proposed system generates the output torque around only one axis. Furthermore, when the rotational axis of the flywheel coincides with the spherical CMG, the system cannot generate any output torque. However, the proposed system can provide a means to overcome this problem if the system is used as an RW, even if the system cannot generate the gyroscopic effect.

Summarizing the above, at least three pairs of omni wheels are required for avoiding singularities when using the spherical CMG as a three-axis attitude control system. As long as the system is driving three pairs of omni wheels, it can generate the gyroscopic effect around two axes orthogonal to the flywheel. If a rotational direction of a spacecraft that agrees with one of the flywheels should be controlled, a pair of omni wheels should drive the sphere around the rotational direction [5e–7].

V. Experiment

In order to verify the effectiveness of the proposed system, we conducted two experiments: verifying a gyroscopic effect as CMG, and generating torque as an RW. The rotational axis of the flywheel is set around the Y-axis. The flywheel is driven using a pair of omni wheels around the X-axis. The moment of inertia of the system was identified experimentally by the bifilar suspension method.

A. Experiment Setup

Figures 8(a)–8(d) show an overview of the proposed system, the sphere containing a flywheel and an IMU to measure the angular velocities of the sphere, an omni wheel, and a flywheel revolving with a constant speed. Tables I and II show the specifications of the flywheel and IMU. The experimental equipment is shown in Fig. 9. In this experiment, we use an air blowing board that blows compressed air up from its surface; thus, it is possible to realize ideally planar motions without friction between the developed model and the blowing board.

B. Result

Figure 10(a) shows the experimental result on the control of angular velocity of the satellite model using the spherical rotor as an RW. It can be calculated from the figure that the angular acceleration of the satellite model $\dot{\omega}_z$ is approximately $1.18\ \mathrm{rad/s^2}$. Therefore, the generated torque as an RW can be calculated using the following equation:

$$\mathbf{T}_\mathrm{RW} = \mathbf{J}\dot{\boldsymbol{\omega}}_B, \tag{20}$$

where $\mathbf{J}$ is the moment of inertia of the satellite model and $\boldsymbol{\omega}_\mathrm{B}$ is the vector of the angular velocity. The moment of inertia I_z is $6.03 \times 10^{-3}\ \mathrm{kg\,m^2}$. From the above equation, the generated torque of RW $\mathbf{T}_\mathrm{RW}$ can be calculated as 7.14×10^{-3} Nm. The results show that the spherical rotor can be used as an RW.

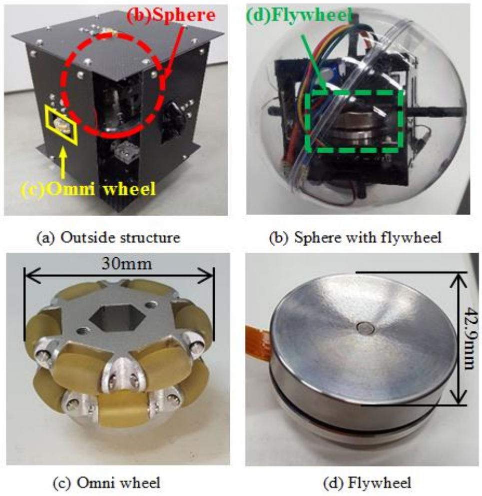


Figure 8. Spherical CMG.

TABLE I. Specifications of the flywheel.

Nominal voltage (V)	12
Max. motor speed (rpmeeee)	2940
Radius (mm)	21.5
Moment of inertia (kg m^2)	92.5×10^{-7}

TABLE II. Specifications of the gyro sensor.

Type	L3GD20
Maximum angular velocity (dps)	250
Resolution of angular velocity (dps)	8.75×10^{-3}

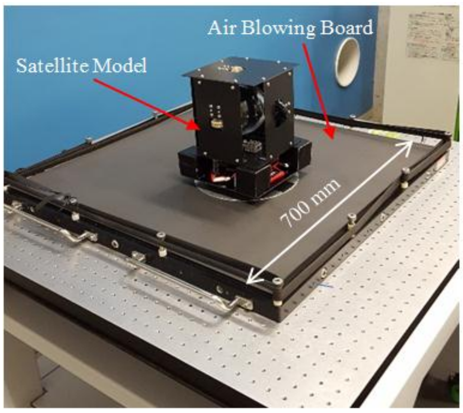


Figure 9. The experiment's equipment for the control of the satellite model using spherical CMG.

Figure 10(b) shows the result of experiment on the control of the satellite model using the spherical CMG. The angular acceleration of the satellite model $\dot{\omega}_z$ is approximately $1.04\ \mathrm{rad/s^2}$, as shown in Fig.10(b). The angular momentum of the satellite $\mathbf{h}_\mathrm{B}$ is expressed as

$$\mathbf{h}_\mathrm{B} = \mathbf{J}\boldsymbol{\omega}_\mathrm{B} + \mathbf{h}_\mathrm{G}, \tag{21}$$

where $\mathbf{h}_\mathrm{G}$ is the angular momentum of the CMG. From the law of conservation of angular momentum, Eq. (21) can be rewritten as follows:

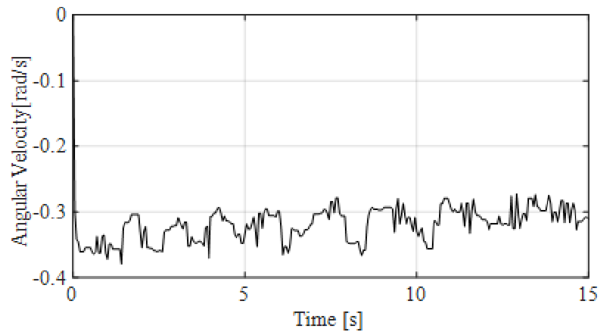
$$\mathbf{J}\dot{\boldsymbol{\omega}}_B + \dot{\mathbf{h}}_G + \boldsymbol{\omega}_\mathrm{B} \times (\mathbf{J}\boldsymbol{\omega}_\mathrm{B} + \mathbf{h}_\mathrm{G}) = 0. \tag{22}$$

Then, the torque $\mathbf{T}_\mathrm{CMG}$ generated by the CMG is expressed as the following equations:

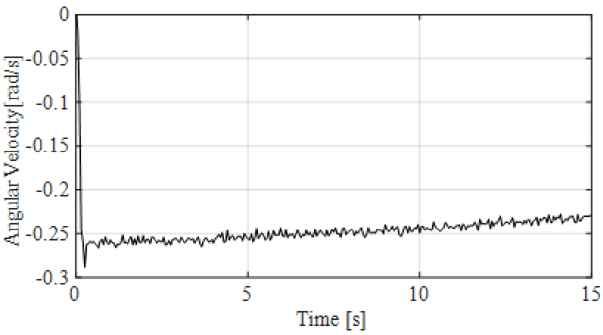
$$-\mathbf{T}_\mathrm{CMG} = \mathbf{J}\dot{\boldsymbol{\omega}}_B + \boldsymbol{\omega}_\mathrm{B} \times \mathbf{J}\boldsymbol{\omega}_\mathrm{B}, \tag{23}$$

$$\mathbf{T}_\mathrm{CMG} = \dot{\mathbf{h}}_G + \boldsymbol{\omega}_\mathrm{B} \times \mathbf{h}_\mathrm{G}. \tag{24}$$

From the experimental results, the moment of inertia of the satellite model around the Z-axis is $I_z = 5.48 \times 10^{-3}\ \mathrm{kg\,m^2}$. The first term on the right-hand side in Eq. (23) is $I_z\dot{\omega}_B = 5.72 \times 10^{-3}\ \mathrm{Nm}$. In this experiment, the second term on the right-hand side in Eq. (23) cannot be calculated because the angular velocities around the X-axis and Y-axis were not obtained. Then, the torque generated by the system was obtained using the maximum rotational speed of the flywheel in Eq. (24). The calculation result is $\mathbf{T}_\mathrm{CMG} = 1.97 \times 10^{-2}\ \mathrm{Nm}$. It is noted that the value of the torque depends on the contact condition between the omni wheel and the sphere.



(a) Time histories of angular velocity of satellite model when varying only angular velocity of spherical shell



(b) Time histories of angular velocity of satellite model when rotating spherical CMG

Figure 10. Experimental results.

VI. Conclusion

In this paper, we proposed a novel attitude control system with a gimballess CMG for a spacecraft. A spherical rotor was used to generate gyroscopic effects. The validity of the proposed system was verified through experiment in terms of the gyroscopic effect around the vertical direction. It was revealed from other experiments that the proposed system can be used as an RW. Further study of three-dimensional attitude control of a satellite model using the proposed system should be conducted.

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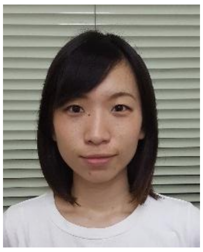

Ayaka Mino obtained her B.E. degree in aerospace engineering from Nihon University, Japan, in 2017. She is currently pursuing an M.E. course at Nihon University. Her research interests include attitude control systems for spacecraft.

Kenji Uchiyama is a Professor of aerospace engineering at Nihon University. He graduated from Tokyo Metropolitan Institute of Technology in 1990 and obtained his doctorate from the institute in 1995. He was also a Visiting Researcher of the University of Strathclyde at Glasgow in 2008. His research interests include the design of controllers for mechanical systems (e.g., unmanned aerial vehicles, space rovers, and spacecraft). He is a Member of the Japan Society for Aeronautical and Space Sciences, the Society of Instrument and Control Engineers, and the American Institute of Aeronautics and Astronautics.