Inverse and Direct Kinematics of Hexa Parallel Robot of Six Degrees of Freedom

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***Abstract*—The description of the kinematics of a par allel robot is based on its structure and geometry, it means, the position and orientation of the platform is analyzed by geometric methods. In addition, it deals with temporary aspects of movement in which the produced forces or torques are not considered. When the particle of a rigid body moves along equidistant trajectories of a fixed plane the body experiences plane movement, classified into three types: translational, rotational and general plane movement; necessary to specify the movement conditions of the active variables that make up a robotic mechanism for a kinematic analysis. Therefore, the present work focuses on the mathematical development of the direct and inverse kinematics of a parallel robot of six degrees of freedom type Hexa, using some strategies such as: spatial decomposition of robot, successive approximations by numerical methods, and Matlab simulations. Results shows the validity of the analysis.**

***Index Terms*—computing methodologies, artificial intelligence, planning and scheduling, robotic planning**

I. Introduction

Kinematics is implemented in robotics for movement analysis which is related to a reference system located in bodies that are moving in space. This analysis focused on the relation between localization variables (position and orientation) of final element and active joint variables [1,2].

The main idea of localization analysis is known the position and orientation of any element of the platform at anytime. There are two kinds of problems: direct kinematics and inverse kinematics [3].

Direct kinematics consists of obtaining position and orientation of final element of mechanisms regarding a reference system from values of active joints and geometry previously established.

Instead, inverse kinematics obtains values of active joints from the localization of final element of robot [4].

Both problems can be solved with some strategies such as: Denavit-Hartenberg (D-H) and geometric solution. D-H is a systematic method based on frames disposition on each robot link hence the homogeneous transform matrix depends on four basic transformation parameters based on distance and rotation angles [5, 6]. Otherwise, geometric solution depends on decomposition of robot spatial geometry in many planar geometric problems in which geometry and trigonometry are used to find the values of the joints variables or platform localization [7, 8]. Parallel robots have many closed loops which hinder D-H application, owing to links with more than one degree of freedom, thus, a transform matrix representation is impossible. As a result, the geometric solution based on vectorial equations is convenient [9]. There exist different ways to solve the inverse kinematics, some leading towards unique solutions [10] [11] whereas others propose different techniques [12].

This work is focused on determining simple direct kinematics and inverse kinematics models of the Hexa robot introduced by Pierrot et. Al. [6] whose structure is based on 6 revolute universal spherical (RUS) limbs which are connected to a mobile platform. This kind of robots has many uses such as: simulators, packing mechanisms and manufacture devices [13].

This paper is organized as follows. First, geometric analysis of Hexa robot. Secondly, inverse kinematics of robot. Thirdly, direct kinematics analyzed with Newton Rhapson method. Finally, the last part concerns about results of simulations and conclusions.

II. Geometric Analysis of Hexa Platform

In order to obtain the mathematical model of a Hexa parallel robot, it is necessary to analyze the structure and geometry in order to know the mobility of the robot and the spatial relationships of the elements that compose it.

The schematic diagram of the analyzed parallel robot is shown in Figure 1 and consists of a base defined by an irregular hexagon and a similar platform that are connected through six arms of two links: input and coupling, with lengths of m and n, respectively.

The links are arranged in an RSS structure, it means that, between them and with the mobile platform the connections are made by spherical joints, while the joint connecting the static platform and the input link is rotational type [14, 15].

A. *Geometry of the Static and Mobile Platform.*

The hexagons that make up the geometry are defined by two magnitudes: the dimensions of the three major edges and the three minor edges, all of them equal to each other. By defining a reference system located in the geometrical center of the hexagon, the six vertices are located as shown in Table I, taking into account that the lengths of the major and minor edges are denoted as a and

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b, respectively; and thus, forming a vector of positions denoted as O_i.

The equations of Table I define the vertices of the mobile platform, only in this case the lengths of the major and minor edges are the same. But it must be taken into account that, being a mobile platform, it is subject to translational and rotational movements defined by the positioning and desired orientation of it. Then must be performed the multiplication of these positions with rotation matrices in α, β, γ, plus the sum of the desired positions in X, Y, Z, according to (1) and (2).

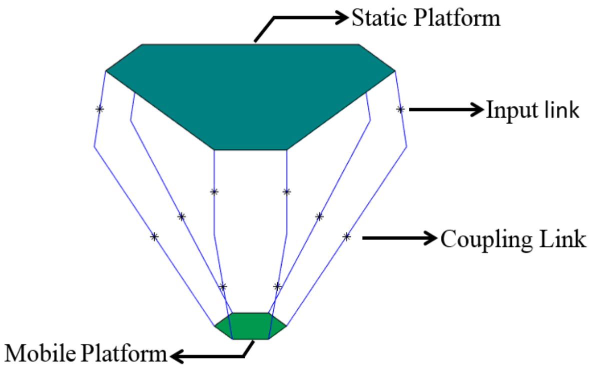

Figure 1. Hexa type platform diagram.

TABLE I. SIMULATION PARAMETERS

Point	X_o	Y_o	Z_o
1	$\frac{\sqrt{3}}{6}(2a+b)$	$\frac{1}{2}(b)$	0
2	$\frac{\sqrt{3}}{6}(2a+b)$	$-\frac{1}{2}(b)$	0
3	$\frac{\sqrt{3}}{6}(a-b)$	$-\frac{1}{2}(a+b)$	0
4	$\frac{\sqrt{3}}{6}(a+2b)$	$-\frac{1}{2}(b)$	0
5	$\frac{\sqrt{3}}{6}(a+2b)$	$\frac{1}{2}(b)$	0
6	$\frac{\sqrt{3}}{6}(a-b)$	$\frac{1}{2}(a+b)$	0

$$R_{\alpha,\beta,\gamma} = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$L_i = R_{\alpha,\beta,\gamma} * p_i + [x \quad y \quad z]^T \quad (2)$$

Where p_i are the points of the mobile platform obtained from Table I.

B. *Inverse Kinematics*

In the previous section, the expressions for the location of the fixed platform in Oi were found as a function of the lengths of the major and minor edges; and for the mobile platform in L_i in terms of the size of the edge that forms the hexagon, in addition to its position and orientation.

Now we proceed with the mathematical analysis of the extremities to find the solution to the inverse kinematic problem. To do this, each arm is located in a coordinate system as shown in Fig. 2, in order to perform a vector analysis.

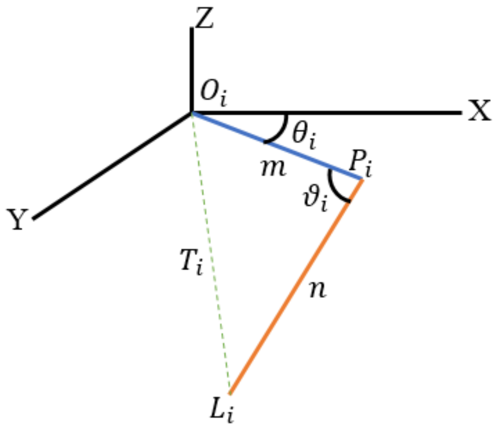

Figure 2. Coordinate System of the Extremities.

Using the geometric of the Fig. 2, we obtain vectorially the expression that defines T_i from the OPL triangle that, being a non-rectangle triangle, its sides are related by the law of the cosine together with the angle ϑ_i and described mathematically in (3). But, since the value of ϑ_i is unknown, the definition of the dot product is applied as shown in (4), to finally obtain (5) that mathematically express T_i.

$$T_i = \sqrt{m^2 + n^2 - 2 \cdot m \cdot n \cdot \cos\vartheta_i} \quad (3)$$

$$\vartheta_i = \cos^{-1}\left(\frac{M_i \cdot N_i}{(m)(n)}\right) \quad (4)$$

$$T_i = \sqrt{m^2 + n^2 - 2(M_i \cdot N_i)} \quad (5)$$

Where, $M_i = O_i - P_i$ y $N_i = T_i - P_i$. In addition, it must be taken into account that the coordinate system for each arm varies, because it must be located in the directions established in Fig. 3, rotated according to the angles specified in (6).

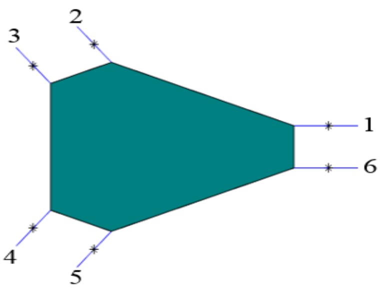

Figure 3. Coordinate System of the Extremities

$$\phi = [0 \quad -60 \quad -60 \quad 60 \quad 60 \quad 0] \quad (6)$$

Then, the vector P_i is obtained, taking into account that since points 1 and 6 are in different quadrants respect to points 2 to 5, two different equations must be considered. For 1 and 6 the value of P_i is described in (7) and for the others in (8).

$$P_i = \begin{bmatrix} m \cdot cos(\theta_i) \cdot cos(\phi_i) + X_o \\ -m \cdot cos(\theta_i) \cdot sin(\phi_i) + Y_o \\ -m \cdot sin(\theta_i) + Z_o]; \end{bmatrix} \quad (7)$$

$$P_i = \begin{bmatrix} -m \cdot cos(\theta_i) \cdot cos(\phi_i) + X_o \\ -m \cdot cos(\theta_i) \cdot sin(\phi_i) + Y_o \\ -m \cdot sin(\theta_i) + Z_o]; \end{bmatrix} \quad (8)$$

For the magnitude of T_i, the mathematics are defined by the Pythagorean theorem as shown in (9).

$$T_i = \sqrt{(X_{L_i} - X_o)^2 + (Y_{L_i} - y_o)^2 + (Z_{L_i})^2} \quad (9)$$

Taking in to account the two equations for T_i in (5) and (9), the terms are equated to find the equation of each arm, adopting the form of (10), according to Fig. 4. Where A_i, B_i and C_i, vary depending of the working point, so (11) are defined for points 1 and 6 and (12) for points 2 to 5.

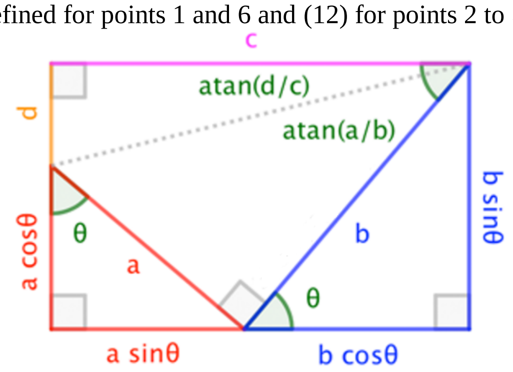

Figure 4. Graphical representation of (10).

$$A_i \sin\theta_i + B_i \cos\theta_i = C_i \quad (10)$$

$$\begin{gathered} A_i = 2 \cdot m \cdot (Z_{L_i} - Z_o) \\ B_i = 2 \cdot m \cdot \left(cos(\phi_i) \cdot (X_o - X_{L_i}) + sin(\phi_i) \cdot (Y_{L_i} - Y_o)\right) \\ C_i = n^2 - m^2 - (X_o - X_{L_i})^2 - (Y_o - Y_{L_i})^2 - (Z_o - Z_{L_i})^2 \end{gathered} \quad (11)$$

$$\begin{gathered} A_i = 2 \cdot m \cdot (Z_{L_i} - Z_o) \\ B_i = 2 \cdot m \cdot \left(cos(\phi_i) \cdot (X_{L_i} - X_o) + sin(\phi_i) \cdot (Y_{L_i} - Y_o)\right) \\ C_i = n^2 - m^2 - (X_o - X_{L_i})^2 - (Y_o - Y_{L_i})^2 - (Z_o - Z_{L_i})^2 \end{gathered} \quad (12)$$

To finally obtain the value of the angle θ_i of each arm, implementing (13).

$$\theta_i = \cos^{-1}\left(\frac{C_i}{\sqrt{A_i^2 + B_i^2}}\right) + \tan^{-1}\left(\frac{A_i}{B_i}\right) \quad (13)$$

C. Direct Kinematics

Direct kinematics is solved base on (14), which determines the length of link in terms of tridimensional positions of points P_i y L_i.

$$n^2 = (X_{L_i} - X_{P_i})^2 + (Y_{L_i} - Y_{P_i})^2 + (Z_{L_i} - Z_{P_i})^2 \quad (14)$$

Applying (14) for each arm of robot, a no-lineal equation system was determined due to X_{L_i}, Y_{L_i} y Z_{L_i} are expresed in position and orientation terms of mobil platform. Hence, there are six unknowns $x, y, z, \alpha, \beta, \gamma$ which are express in 6x6 equation system showed in (15).

$$f_i = (X_{L_i} - X_{P_i})^2 + (Y_{L_i} - Y_{P_i})^2 + (Z_{L_i} - Z_{P_i})^2 - n^2 = 0 \quad (15)$$

Previous system has a unique solution and it can be resolved with numeric methods, for instance, Newton Raphson multivariable method. That method requires Jacobian Matrix [13] which is expressed in (16).

$$\begin{gathered} f'_{xi} = \frac{\partial f_i}{\partial x}, \quad f'_{yi} = \frac{\partial f_i}{\partial y}, \quad f'_{zi} = \frac{\partial f_i}{\partial z} \\ f'_{\alpha i} = \frac{\partial f_i}{\partial \alpha}, \quad f'_{\beta i} = \frac{\partial f_i}{\partial \beta}, \quad f'_{\gamma i} = \frac{\partial f_i}{\partial \gamma} \end{gathered} \quad (16)$$

Newton Rhapson method is implemented based on (17), objective function is related to (18).

$$[x, y, z, \alpha, \beta, \gamma]_{i+1} = [x, y, z, \alpha, \beta, \gamma] - \frac{f_i}{f'_i} \quad (17)$$

$$\begin{aligned} error = ((X - X_d)^2 + (Y - Y_d)^2 + (Z - Z_d)^2 \\ + (\alpha - \alpha_d)^2 + (\beta - \beta_d)^2 \\ + (\gamma - \gamma_d)^2)^{1/2} \leq tol \end{aligned} \quad (18)$$

If the determinant of Jacobian matrix is equal zero, the matrix will be singular, and its inverse cannot be calculated. This case describes that robot loses mobility or cannot reach the required position because of its geometry. Therefore, its important calculate the inverse matrix following (19) [16, 17].

$$\frac{dq}{dt} = f'(q) \cdot \dot{q} \quad (19)$$

Where q denoted generalized coordinates: x, y, z, α, β, γ.

III. Results and Discussions

The simulation of the previously established calculations is performed with the values specified in Table II. And we start from a resting position for x, y, z, in meters α, β, γ in degrees of $[0, 0, -0.45, 0, 0, 0]$ of the robot as shown in Fig. 5.

TABLE II: Simulation Parameters

Parameter	Value
m	0.15 m
n	0.35 m
a	0.3 m
b	0.1 m
c	0.05 m

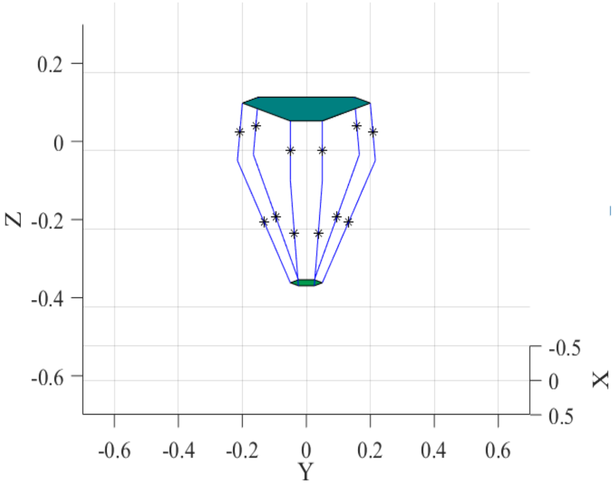

Figure 5. Resting position of Hexa robot

From the above, we obtain different trajectories of the robot to verify the correct calculus of the direct and inverse kinematics, as seen in Figs. 6 for x, y, z, α, β, γ from [0,0.5,-0.25,45,20,0], Fig. 7 to x, y, z, α, β, γ from [0,-0.5,-0.2,45,20,30] and Fig. 8 to x, y, z, α, β, γ from [0,-0.1,-0.35,-45,-45,10].

IV. Conclusions

The Hexa parallel robot has advantages over typical architectures such as delta configuration, since with this one can have control over the three degrees of position and three orientation, which makes it viable for the implementation of any type of work where both speed and accuracy are required, these being achieved after the implementation of closed control loops.

On the other hand, the kinematic analysis of the parallel robotic platform is completely necessary for the calculation of the differential kinematics, which is crucial to perform a kinematic control, in addition giving the guidelines to perform the dynamic analysis and dynamic control of the platform.

As future work, the implementation of control loops will be implemented to obtain the desired behaviors on physical platforms.

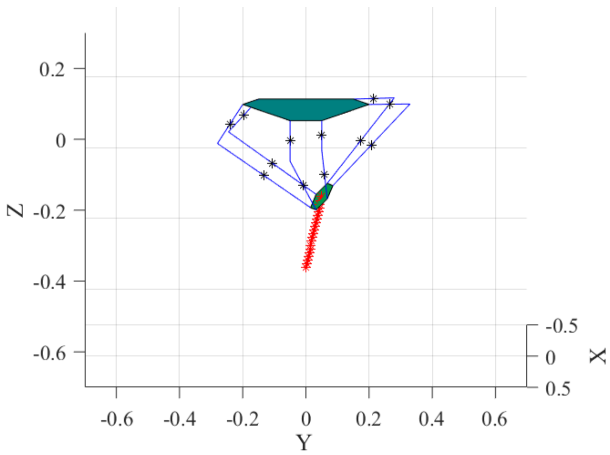

Figure 6. Hexa robot in position 1

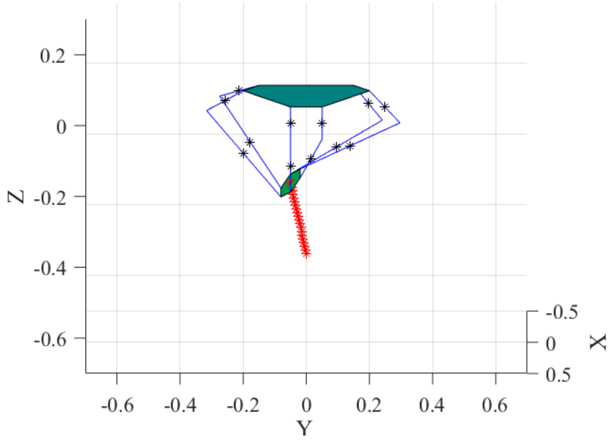

Figure 7. Hexa robot in position 2

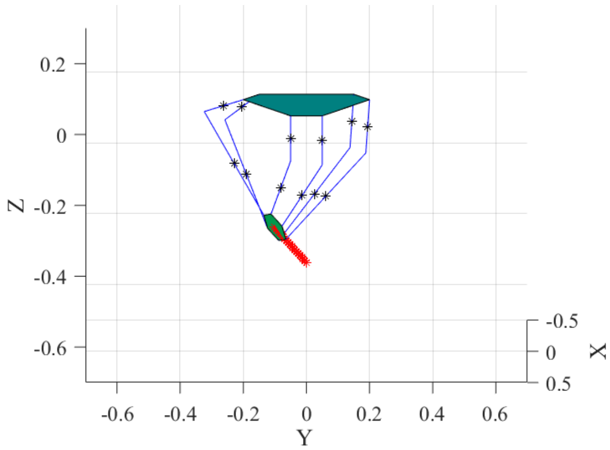

Figure 8. Hexa robot in position 3

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References

[1] H. A. Alashqat, "Modeling and high precision motion control of 3 dof parallel delta robot manipulator," The Islamic University of Gaza, Faculty of Engineering, Electrical Department, Master Thesis, 2013.

[2] E. Mirshekari, A. Ghanbarzabeh, K. H. Shirazi, "Structure comparison and optimal design of 6-rus parallel manipulator based on kinematic and dynamic performances," *Latin American Journal of Solids and Structures*, vol. 13, pp. 2414 – 2438, 2016.

[3] A. Olsson, "Modelling and control of a delta 3 robot" Department of Automatic Control, Lund University, 2009.

[4] P. Zamudio, V. Gonzalez, M. Parra, A. Ramirez, "Cinemática Diferencial de un Manipulador Paralelo Plano 3RRR-(RRR) con Actuación Virtual Indirecta," *Ingeniería mecánica – Tecnologia y Desarrollo*, vol. 6, no. 3, pp 321 – 331, 2016.

[5] J. Vázquez, F. Cuenca, "Cinemática Inversa y Análisis Jacobiano del Robot Paralelo Hexa," *Memorias Del Xv Congreso Internacional Annual De La Somim*, vol. 1, no. 4, pp 800-810, Sonora. México, 2009.

[6] F. Pierrot, P. Dauchez, A. Fournier, "HEXA: a fast six-DOF fully-parallel robot," *Fifth International Conference on Advanced* Robotics '*Robots in Unstructured Environments*, 1991.

[7] J. C. John, "Introduction to robotics: Mechanics & control," Boston, Addison-Wesley Publishing Company, 1986.

[8] J. Sabater, "Control de Robots". División de Ingeniería de Sistemas y Automática del Grupo de Tecnología Industrial de la Universidad Miguel Hernández (UMH) Campus Elche, 2004.
[9] R. Cisneros, "Modelo Matemático de un Robot Paralelo de Seis Grados de Libertad," *National Institute of Advanced Industrial Science and Technology*, Santa Catarina Mártir, Cholula, Puebla, 2006.
[10] C. Vaida, D. Pisla, F. Covaciu, B. Gherman, A. Pisla, and N. Plitea, "Development of a control system for a HEXA parallel robot," *IEEE International Conference on Automation, Quality and Testing, Robotics (AQTR)*, pp. 1-6, 2016.
[11] B. Mohammed, "Planning a trajectory of a 6-DOF parallel robot HEXA," *Electrical and Information Technologies (ICEIT) International Conference*, pp. 300-305, 2016.
[12] A. Zubizarreta, M. Larrea, E. Irigoyen, I. Cabanes, "Real time parallel robot direct kinematic problem computation using neural networks," in *Proc. 10th International Conference on Soft Computing Models in Industrial and Environmental Applications, Advances in Intelligent Systems and Computing*, vol. 368, pp. 285-295, 2015.
[13] S. Tartari, E. Lustosa, "Kinematics and workspace analysis of a parallel architecture robot: The Hexa," *ABCM Symposium Series in Mechatronics*, vol. 2, pp 158-165, 2006.
[14] R. C. Hibbeler, "Mecánica vectorial para ingenieros: Dinámica," Décima edición. México, Pearson Educación, 2004.
[15] T. Yukio, F. Hiroaki, K. Masafumi, H. Kazuya, "6-RSS type spatial in-parallel actuated mechanism with six degrees of freedom," Department of Mechanical Engineering and Science: Tokio Institute of Technology, 1998.
[16] W. S. Mark and M. Vidyasagar, *Robot Dynamics and Control*, New York, John Wiley & Sons, 1989.
[17] L. W. Tsai, *Robot Analysis: The Mechanics of Serial and Parallel Manipulators*, Maryland, John Wiley & Sons, Inc., 1999.

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