Finding Area of Desired Control Quality of an Unmanned Underwater Vehicle Motion in a Plane of Its Construction Parameters

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***Abstract*—The paper is dedicated to solving a problem of finding the area of values of unmanned underwater vehicle construction parameters providing desired oscillability degree of its motion control system with interval parameters. Construction parameters such as unmanned underwater vehicle length and maximal diameter, which are primary parameters, are linked with secondary parameters of unmanned underwater vehicle motion control system included in coefficients of its interval characteristic polynomial such as added masses of water and hydrodynamic coefficients. Authors propose a method of estimating unmanned underwater vehicle construction parameters by finding intervals of characteristic polynomial coefficients providing desired motion control quality. Example of the proposed method application is also provided.**

***Index Terms*—construction parameters, interval parameters, oscillability degree, interval characteristic polynomial, unmanned underwater vehicle, sixth term**

I. Introduction

Developing unmanned underwater vehicles (UUV) and their motion control systems [1] – [7] requires estimating UUV construction parameters, such as UUV length and UUV maximal diameter. These parameters are not included in UUV motion control system model directly, but only as a part of added masses and hydrodynamic coefficients. Let us now designate length and diameters as primary parameters, added masses and hydrodynamic coefficients – as secondary parameters. Secondary parameters dependences on primary parameters are described via certain expressions. Secondary parameters are included in coefficients of interval characteristic polynomial of the system considered. Consequently, values of primary parameters can be found through estimating coefficients of characteristic polynomial and finding values of secondary parameters on their basis.

For each of values of primary parameters, nominal values of secondary parameters can be found. Then, on a basis of sector stability criterion, a linear controller can be synthesized. Let us assume, that controller synthesis resulted in a characteristic polynomial, whose coefficients (except m, $m \geq 2$ nearest-neighbor lower coefficients) include secondary parameters of the system. As far as the aim is to find values of secondary parameters, they will be considered as uncertain ones varying within some intervals, which can be expressed from coefficients of characteristic polynomial. Let us designate a vector of secondary parameters as T and write a characteristic polynomial as follows:

$$A(s) = \left[a_n(\vec{T})\right]s^n + \left[a_{n-1}(\vec{T})\right]s^{n-1} + \cdots \left[a_m(\vec{T})\right]s^m + a_{m-1}s^{m-1} + \ldots + a_0. \quad (1)$$

Providing stability of (1) and its acceptable oscillability degree [8] – [14] consists in developing such characteristic polynomial, whose roots will be placed within desired sector on the left half of a complex plane. To do this, it is proposed to use an algebraic method based on coefficient indices of stability and oscillability [15], which can be expressed through coefficients of characteristic polynomial.

The main aim of the research is to develop a method of finding an area of values of UUV construction parameters, within of which desired motion stability and oscillability are guaranteed despite interval parametric uncertainty of the system. To reach the aim formulated, it is proposed to accomplish several objectives:

1. Derive a characteristic polynomial (1) of UUV motion control system. Deriving such polynomial consists in finding such interval coefficients $\left[a_i\right]$, $i = \overline{m, n}$, which provide desired oscillability degree of the system.

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2. On a base of dependencies $a_i(\vec{T})$ and intervals $[a_i]$, $i=\overline{m,n}$ find an area of acceptable oscillability degree in a space of secondary parameters.
3. Obtain the area of primary parameters on a base of area of secondary parameters.

II. Deriving an Interval Characteristic Polynomial of the System with Desired Sector Stability

Let us consider linear time-invariant continuous control system, whose characteristic polynomial is written as follows:

$$A(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_0 \tag{2}$$

Stability index λ_i depending on each four nearest-neighbor coefficients of (2) must be introduced as follows:

$$\lambda_i = a_{i-1}a_{i+2} / \left(a_i a_{i+1}\right), \quad i=\overline{1,n-2}. \tag{3}$$

According to [14], to provide stability of the system with characteristic polynomial (2) it is enough to satisfy following inequality:

$$\lambda_i < 0.465, \quad i=\overline{1,n-2}. \tag{4}$$

Oscillability index δ_i also should be introduced as follows:

$$\delta_\mathrm{i} = \frac{a_i^2}{a_{i-1}a_{i+1}}, \quad i=\overline{1,n-1}. \tag{5}$$

It was determined [14], that in order to place roots of (1) within desired sector $\pm\varphi \ (\varphi \le \frac{\pi}{2})$ following inequality should be satisfied.

$$\delta_i \ge \delta_o(n,\varphi), \quad i=\overline{1,n-1}, \tag{6}$$

where δ_o can be found from the diagram in the Fig. 1.

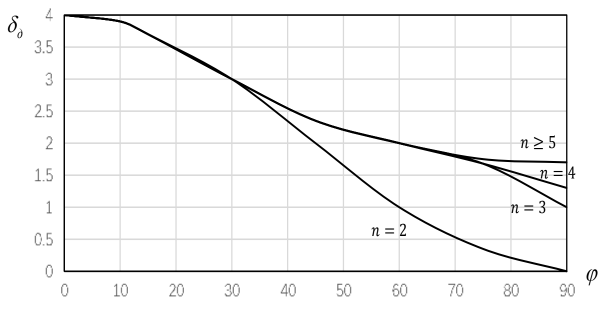


Figure 1. Diagram of δ_o values depending on φ for polynomials of different order

Let us now develop an interval extension of expressions for stability index λ_i and oscillability index δ_i

$$D(s) = \left[a_n\right]s^n + \left[a_{n-1}\right]s^{n-1} + \cdots + \left[a_0\right], \tag{7}$$

where $\underline{a_i} \le a_i \le \overline{a_i}$; $\underline{a_i}$ - minimal value of a_i ; $\overline{a_i}$ - maximal value of a_i .

According to (3) and (4) to provide stability of ICP (7) it is enough to satisfy following inequalities:

$$\overline{\lambda_i} = \frac{\overline{a_{i-1}}\,\overline{a_{i+2}}}{\underline{a_i}\,\underline{a_{i+1}}} \le 0,465, \quad i=\overline{1,n-2}. \tag{8}$$

According to (5) and (6), to place ICP roots in a desired sector it is enough to satisfy following inequality:

$$\underline{\delta_i} = \frac{\underline{a_i}^2}{\overline{a_{i-1}}\,\overline{a_{i+1}}} \ge \delta_o, \quad i=\overline{1,n-1}. \tag{9}$$

Let us assume, that in characteristic polynomial (7), whose order is higher than 3, values of at least two lower coefficients are known. Now it is necessary to find interval of other coefficients values, which include secondary parameters of the system and provide allocation of ICP roots in a desired sector.

On a base of (8) and (9), let us derive a system of inequalities for a polynomial (1):

$$\begin{cases} \dfrac{\overline{a_{i-1}}\,\overline{a_{i+2}}}{\underline{a_i}\,\underline{a_{i+1}}} \le 0,465, \quad i=\overline{1,n-2} \\ \dfrac{\underline{a_i}^2}{\overline{a_{i-1}}\,\overline{a_{i+1}}} \ge \delta_o, \quad i=\overline{1,n-1}. \end{cases} \tag{10}$$

Let us notice, that inequalities system (10) includes inequalities with constant or uncertain lower coefficients for $m \ge 3$. These inequalities must be checked before solving a whole system (10).

On a base of analyzing expressions from system (10), an algorithm of finding desired limits of unknown ICP (7) coefficients was developed. It include their subsequent calculation from lower ones to higher ones and includes following steps:

1. Defining index I of the lower uncertain coefficient ($i \ge 2$).
2. Defining acceptable values of oscillability index δ_o on a base of diagram in the Fig. 1.
3. Finding $\overline{a_i} = \dfrac{\overline{a_{i-1}}^2}{\delta_o \underline{a_{i-2}}}$.
4. Composing a system of two inequalities

$$\begin{cases} \overline{a_{i+1}} < \dfrac{0,465 \cdot \underline{a_{i-1}}\,\underline{a_i}}{\overline{a_{i-2}}}; \\ \overline{a_{i+1}} < \dfrac{\underline{a_i}^2}{\delta_o \overline{a_{i-1}}}, \end{cases}$$

where unknown variables are $\underline{a_i}$ and $\overline{a_{i+1}}$.

5. Solving the inequalities system for $0<\underline{a_i}<\overline{a_i}$ and choosing values $\underline{a_i}$ and $\overline{a_{i+1}}$ from the solution.
6. Increasing coefficient index $i=i+1$. Steps 4 and 5 must be repeated while $i<n$.
7. If $i=n$ then limits of all ICP (7) coefficients are found besides $\underline{a_n}$. It should be defined from the range $0<\underline{a_n}<\overline{a_n}$.

III. Finding Areas of Primary and Secondary Parameters of the System Providing Desired Sector Stability

The research resulted in a method of finding an area of acceptable values of control system parameters providing desired oscillability. The method includes following steps:

1. Finding intervals of ICP coefficients $[a_i]=[\underline{a_i};\overline{a_i}]$, which provide ICP stability and desired oscillability degree.
2. Composing a system of double inequalities on a base of ICP coefficients dependencies from secondary parameters and previously obtained limits of ICP coefficients:

$$\underline{a_i}<a_i(\vec{T})<\overline{a_i},\ i=\overline{m,n}. \quad (11)$$

3. Solving inequalities system (11) and plotting an area of acceptable values of the systems secondary parameters $\vec{T}$.
4. Plotting the area of primary parameters of the system on a base of certain expressions linking them with secondary parameters.

To obtain the area of primary parameters from the area of secondary parameters, expression linking the together should be used. In our case, expressions linking added mass of water λ_{11} and hydrodynamic coefficient of drag force c_x with length l and diameter D of UUV must be obtained.

Estimate of drag force hydrodynamic coefficient can be calculated via following expression [16] - [18]:

$$c_x=k\cdot(c_f+\Delta\xi_{\text{chr}}+c_\phi+c_x^{hs}), \quad (12)$$

where k –coefficient of UUV hull curvature; c_f, c_ϕ – hydrodynamic coefficients of flat plate and UUV hull shape; $\Delta\xi_{chr}$ –correction on a UUV hull roughness; c_x^{ha} – hydrodynamic coefficient of UUV hull appurtenances.

It should be noticed, that all summands in (12) depend on UUV size. For example, coefficient of UUV hull curvature k depends on relative lengthening of UUV and, consequently, on it diameter and length. Hydrodynamic coefficients c_f and c_ϕ of flat plate and UUV hull shape depend on Reynolds number and, consequently, on UUV length. Assuming that on early steps of UUV development there is now information about UUV hull appurtenances, let us ignore their hydrodynamic coefficient c_x^{ha}. Also, let us consider UUV surface smooth enough to ignore roughness correction $\Delta\xi_{chr}$.

Let us use the following formula of added mass in following calculations:

$$\lambda_{11}=\pi\cdot\rho\cdot\frac{D^2}{4}, \quad (13)$$

where ρ –water density; D –maximal diameter of UUV.

Finally, the fourth step of the method proposed (transfer from are of secondary parameters to area of primary parameters) will be performed on a base of expressions (12) and (13).

IV. Example

Let us consider a characteristic polynomial of the forward motion of the unmanned underwater vehicle:

$$D(s)=a_5\,s^5+a_4\,s^4+a_3\,s^3+a_2\,s^2+a_1\,s+a_0, \quad (14)$$

where

$a_5=2T_ST_T(m+\lambda_x)$;

$a_4=2(T_Sm+T_Tm+T_S\lambda_x+T_T\lambda_x)+c_xT_ST_TV^{2/3}\rho\,\text{k}$;

$a_3=2(m+\lambda_x)+c_xT_SV^{2/3}\rho k+c_xT_TV^{2/3}\rho k$;

$a_2=c_xV^{2/3}\rho\,\text{k}$; $a_1=4K_PK_TK_S$; $a_0=4K_IK_TK_S$.

Coefficients of (14) include following parameters of a UUV: $T_S=0.01\,s$ – time constant of a sensor; $T_T=1\,s$ – time constant of a thruster; $m=1140\,kg$ – UUV mass; $V=11\,m^3$ – water displacement of a UUV; $\rho=1000\,kg/m^3$ – water density; $k=1$ – linearization coefficient; $K_P=0.7$ and $K_I=0.03$ – parameters of PI-controller; $K_T=50$ – transfer coefficient of a thruster; $K_S=1$ – transfer coefficient of a sensor. Let us notice, that λ_x – added mass of water and c_x – hydrodynamic coefficient of drag force are secondary to parameters determining UUV size: diameter D and length L, which are primary parameters.

Parameters λ_x and c_x are not included in two lower coefficients of characteristic polynomial. There is a problem of finding an area of values of these two parameters providing desire stability degree and oscillability degree of the system considered. To do this, all coefficients including λ_x and c_x will be considered uncertain and interval. Considering this, interval characteristic polynomial (14) can be rewritten as follows:

$$D(s)=[a_5]s^5+[a_4]s^4+[a_3]s^3+[a_2]s^2+241s+9.2. \quad (15)$$

On a base of (8) and (9) let us derive a system of inequalities (10) for polynomial (15). Solution of this system gives interval values of (15) coefficients: $[a_5]=[100;500]$; $[a_4]=[3000;5000]$; $[a_3]=[6000;9000]$; $[a_2]=[2000;3100]$.

Then, on a base of dependencies of these coefficients and parameters λ_x and c_x, a system of inequalities (11) with two variables can be derived. Solution of this system is shown in the Fig. 2.

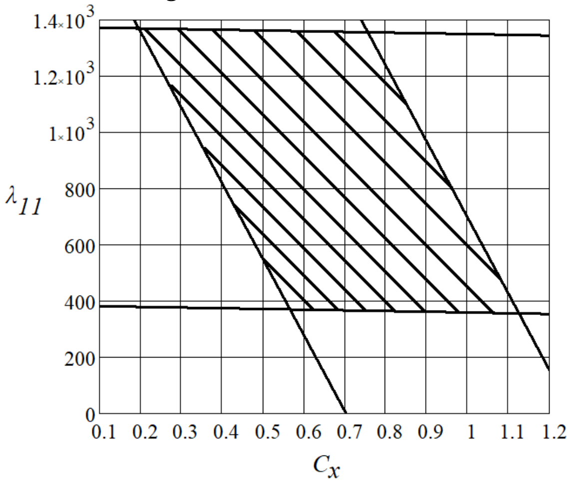
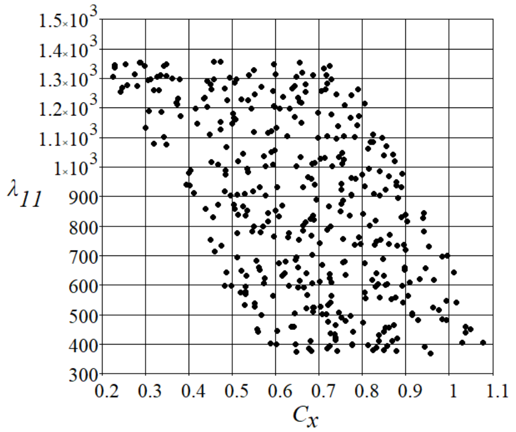


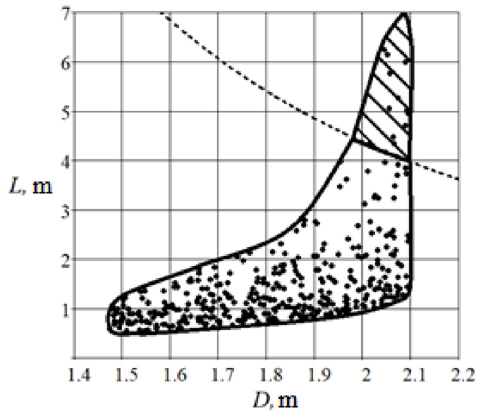
Figure 2. Values area of secondary parameters

Aforementioned calculations resulted in the area of values of secondary parameters λ_x and c_x. To plot this area in coordinates of UUV length and diameter, a Monte-Carlo method [19], [20] and expressions (12) and (13) were used. First, with the help of Monte-Carlo method an image of the area shown in the Fig. 2 was built (see Fig. 3(a)). Then, with the help of (12) and (13) coordinates of each dot within the source domain were recalculated and plotted in a plane of UUV diameter and length (see Fig. 3(b)).

Dotted line in the Fig. 3b shows the set of values of UUV diameter and length, which provide desired volume inside UUV hull and water displacement. So, all calculations performed via proposed method resulted in hatched area shown in the Fig. 3b. Every combination of UUV diameter and length chosen within the hatched area will provide desired control quality and UUV motion dynamics. It should be noticed, that on the late steps of development UUV dynamics can be improved by proper controller synthesis.



a)



b)

Figure 3. Area of parameters values in a plane of (a) secondary parameters and (b) primary parameters of UUV

V. Conclusion

Considered research resulted in a method of estimating primary parameters of control system on a base of desired control quality. An example of finding area of values of UUV length and diameter on a base of desire stability degree and oscillability degree of its motion control system with interval parameters was provided.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Gayvoronkiy Sergey find area of values of UUV length and diameter on a base of desire stability degree and oscillability degree of its motion control system with interval parameters, wrote the paper.

Ezangina Tatiana developed a method of finding an area of values of UUV construction parameters and wrote the paper.

Khozhaev Ivan calculated a characteristic polynomial of the forward motion of the unmanned underwater vehicle.

Nesenchuk Alla analyzed the data.

All authors had approved the final version.

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