Enhancing Understanding of Changes in Solution Rankings with Variations in User Coefficients in AROMAN Method

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***Abstract*—Alternative Ranking Order Method Accounting for Two-Step Normalization (AROMAN) is a recently discovered method for Multi-Criteria Decision Making (MCDM). When using it, decision-makers must choose the values of two parameters, one being the value of the synthetic coefficient of normalized values (β), and the other being the value of the correlation coefficient between the quantities of criteria in each form (λ), referred to as user coefficients. Both of these coefficients have values ranging from 0 to 1. In previous AROMAN applications, both coefficients were consistently chosen as 0.5. This study investigates the impact of selecting different values for these coefficients. The investigation was carried out by varying the values of β and λ from 0.1 to 0.9. Results showed that the rankings of options were minimally affected by β and λ values. Notably, the best option consistently remained independent of the values chosen for β and λ.**

***Keywords*—Multi-Criteria Decision Making (MCDM), Alternative Ranking Order Method Accounting for Two-Step Normalization (AROMAN) method, user coefficients**

I. INTRODUCTION

Among the over 200 Multi-Criteria Decision Making (MCDM) methods proposed by researchers [1], Alternative Ranking Order Method Accounting for Two-Step Normalization (AROMAN) is one of the newest, discovered in April 2023 [2]. A unique aspect of AROMAN is its simultaneous use of two different data normalization methods. While most MCDM methods use a single normalization method such as Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) [3], Vlsekriterijumska optimizacijaI KOmpromisno Resenje (VIKOR) [4], Measurement Alternatives and Ranking according to COmpromise Solution (MARCOS) [5], Multi-Attributive Border Approximation area Comparison (MABAC) [6], etc., While others do not use any normalization method like Faire Un Choix Adéquat (FUCA) [7], Collaborative Unbiased Rank List Integratio (CURLI) [8], etc. Due to this difference, when using the AROMAN method, users must choose the values of two coefficients, referred to as user coefficients. The first coefficient is the synthesis coefficient of normalized values, denoted as β. This is a coefficient for synthesizing normalized values when normalized by two methods, including linear normalization and vector normalization. The second coefficient is the correlation coefficient between the quantities of criteria in each form (larger criteria are better, and smaller criteria are better), denoted as λ. This is a coefficient reflecting the preference for finding the best alternative that leans towards alternatives with more significant criteria or towards alternatives with smaller criteria. The specific usage of these coefficients will be clarified in the next chapter of this study. According to AROMAN proponents, both coefficients have values between 0 and 1. Despite its recent introduction, AROMAN has been applied in various recent studies, such as electric vehicle selection [9], bicycle type selection in delivery logistics [10], human resource management strategy selection [11], and assessing the sustainable competitive position in the economic development of Turkey compared to neighboring countries [12]. If β is chosen to be less than 0.5, it means prioritizing the normalized ratio according to the vector method over the linear normalization method, and vice versa. If λ is chosen to be less than 0.5, it means ranking options will prioritize those with fewer criteria as better. Conversely, if λ is greater than 0.5, ranking options will prioritize those with more criteria as better. This implies that the choice of values for β and λ will heavily depend on the decision-maker's perspective. This highlights that in the context of many differences among MCDM methods, AROMAN not only provides flexibility but also raises new questions about the role of decision-makers in shaping outcomes. However, all studies applying this method have chosen both coefficients β and λ as 0.5. What happens if these coefficients are selected as different values? This question prompted the authors of this article to seek answers in this study. The uniqueness of this research lies in expanding understanding of AROMAN,

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not only from a technical standpoint but also from the user's perspective. The steps to apply the AROMAN method are outlined in Section II. The investigation into the impact of the values of β and λ on the rankings of options is presented in Section III of this article, where the investigation was conducted in two different domains: robotic welding selection and cutting oil selection. The novel findings discovered in this study constitute the final part of this article.

II. The AROMAN Method

To rank options using the AROMAN method, the following steps need to be sequentially applied [2].

Step 1: If there are *m* options to be ranked, each with *n* criteria, construct a decision matrix with *m* rows and *n* columns. Let x_{ij} be the value of criterion *j* for option *i*, where $j = 1$ to *n*, $i = 1$ to *m*. The weight of criterion *j* is denoted as w_j. Criteria with larger values indicating better performance are labeled as type *B*, while those with smaller values indicating better performance are labeled as type *C*.

Step 2: Standardization of data is done in two ways: 1) linear normalization method, and 2) vector normalization method.

$$t_{ij} = \frac{x_{ij} - minx_{ij}}{maxx_{ij} - minx_{ij}} \tag{1}$$

$$t_{ij}^{*} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}} \tag{2}$$

Step 3: Calculate the average normalized value according to Eq. (3), where β is the synthesis coefficient of normalized values, with values ranging from 0 to 1 [2].

$$t_{ij}^{norm} = \frac{\beta \cdot t_{ij} + (1-\beta) \cdot t_{ij}^{*}}{2} \tag{3}$$

Step 4: Calculate the average normalized value considering the weights of the criteria according to Eq. (4).

$$\widehat{t_{ij}} = t_{ij}^{norm} \cdot w_j \tag{4}$$

Step 5: Sum the normalized values considering the weights for the criteria according to Eqs. (5) and (6).

$$L_i = \sum_{j=1}^{n} \widehat{t_{ij}}^{(min)} \quad \text{if } j \in C \tag{5}$$

$$A_i = \sum_{j=1}^{n} \widehat{t_{ij}}^{(max)} \quad \text{if } j \in B \tag{6}$$

Step 6: Calculate the scores of the options according to Eq. (7), where λ is the correlation coefficient between the quantities of type *B* and type *C* criteria, with values ranging from 0 to 1 [2].

$$R_i = L_i^{\lambda} + A_i^{(1-\lambda)} \tag{7}$$

Step 7: The option with the highest score R_i is the best option, and vice versa.

III. Results and Discussion

In this chapter, the investigation into the influence of the values of β and λ on the rankings of options is conducted in two different domains. The first case involves ranking various types of robotic welding, while the second case involves ranking different cutting oils.

A. Case 1

Seven types of welding robots were considered for ranking, denoted as WR1, WR2, WR3, WR4, WR5, WR6, and WR7. Data on these welding robots were referenced from a published study [13]. Six parameters were chosen as criteria to describe each option, including HR: Horizontal reach (mm), VR: Vertical reach (mm), PR-Cost (USD), ER: Error (% mm), LC: Load capacity (kg), and NoP-Number of poles. HR, VR, LC, and NoP are type *B* criteria, while PR and ER are type *C* criteria. The weights of these six criteria were calculated using the *MEREC* (Method based on the Removal Effects of Criteria) method. All information about the seven welding robots was synthesized in Table I. The Measurement Alternatives and Ranking according to COmpromise Solution (MARCOS) and Preference Selection Index (PSI) methods were also used to rank these welding robots in a previous study [13]. In addition, two methods, *T*OPSIS and Root Assessment Method (RAM), have also been used to rank options in this case. TOPSIS is chosen because it is evaluated as the most widely used method among MCDM methods [14]. The data normalization method used in TOPSIS is the vector method. The steps for applying the TOPSIS method can be found in recent studies [15, 16]. RAM is also used because it is a very new method. RAM provides different compensations between benefit and cost criteria, which is rare in MCDM methods. The data normalization method used in this method is linear sum. The steps to apply the RAM method can be found in [17].

The ranking results of robot types using four methods, MARCOS, PSI, TOPSIS, and RAM, will be used to compare with the ranking results using the AROMAN method in this article.

The ranking of welding robots using the AROMAN method was performed as follows. In the first scenario, the values of both β and λ were chosen as 0.5.

The decision matrix is the table containing information about the welding robots (Table I).

Normalization of data using two methods through the application of Eqs. (1) and (2) was performed, and the results were synthesized in two, Tables II and III.

TABLE I. TYPES OF WELDING ROBOTS [13]

Welding Robot	HR	VR	PR	ER	LC	NoP
WR1	727	1312	6809	4%	8	6
WR2	927	1693	6170	6%	7	6
WR3	1440	2511	4213	12%	12	6
WR4	1730	3089	5532	12%	25	6
WR5	2010	3649	6000	16%	12	6
WR6	3121	5616	5319	16%	6	6
WR7	1434	2475	3553	16%	3	7
Weight	0.2594	0.2554	0.0931	0.1775	0.2067	0.0079

TABLE II. VALUES OF t_{ij}

Welding Robot	HR	VR	PR	ER	LC	NoP
WR1	0.00000	0.00000	1.00000	0.00000	0.22727	0.00000
WR2	0.08354	0.08852	0.80375	0.16667	0.18182	0.00000
WR3	0.29783	0.27858	0.20270	0.66667	0.40909	0.00000
WR4	0.41896	0.41287	0.60780	0.66667	1.00000	0.00000
WR5	0.53592	0.54298	0.75154	1.00000	0.40909	0.00000
WR6	1.00000	1.00000	0.54238	1.00000	0.13636	0.00000
WR7	0.29532	0.27021	0.00000	1.00000	0.00000	1.00000

TABLE III. VALUES OF t^*_{ij}

Welding Robot	HR	VR	PR	ER	LC	NoP
WR1	0.15398	0.15525	0.47020	0.12017	0.24445	0.36858
WR2	0.19634	0.20034	0.42608	0.18025	0.21390	0.36858
WR3	0.30500	0.29713	0.29093	0.36051	0.36668	0.36858
WR4	0.36642	0.36553	0.38202	0.36051	0.76392	0.36858
WR5	0.42572	0.43180	0.41434	0.48067	0.36668	0.36858
WR6	0.66104	0.66456	0.36731	0.48067	0.18334	0.36858
WR7	0.30372	0.29287	0.24536	0.48067	0.09167	0.43001

The average normalized value was calculated according to Eq. (3) and the data are presented in Table IV. It should be noted that in this case, calculations were carried out with β chosen as 0.5.

Eq. (4) was applied to calculate the average normalized value considering the weights of the criteria, resulting in Table V.

TABLE IV. NORMALIZED AVERAGE VALUES t^{norm}_{ij}

Welding Robot	HR	VR	PR	ER	LC	NoP
WR1	0.03850	0.03881	0.36755	0.03004	0.11793	0.09214
WR2	0.06997	0.07221	0.30746	0.08673	0.09893	0.09214
WR3	0.15071	0.14393	0.12341	0.25679	0.19394	0.09214
WR4	0.19635	0.19460	0.24745	0.25679	0.44098	0.09214
WR5	0.24041	0.24370	0.29147	0.37017	0.19394	0.09214
WR6	0.41526	0.41614	0.22742	0.37017	0.07993	0.09214
WR7	0.14976	0.14077	0.06134	0.37017	0.02292	0.35750

TABLE V. VALUES OF $\widehat{t_{ij}}$

Welding Robot	HR	VR	PR	ER	LC	NoP
WR1	0.00999	0.00991	0.03422	0.00533	0.02438	0.00073
WR2	0.01815	0.01844	0.02862	0.01539	0.02045	0.00073
WR3	0.03909	0.03676	0.01149	0.04558	0.04009	0.00073
WR4	0.05093	0.04970	0.02304	0.04558	0.09115	0.00073
WR5	0.06236	0.06224	0.02714	0.06570	0.04009	0.00073
WR6	0.10772	0.10628	0.02117	0.06570	0.01652	0.00073
WR7	0.03885	0.03595	0.00571	0.06570	0.00474	0.00282

Values L_i, A_i, and R_i were calculated using Eqs. (5)–(7), and the results were synthesized in Table VI. It should also be reiterated that when applying Eq. (7), the value of λ was chosen as 0.5. In the last column of this table, the ranking of the welding robots based on their R_i values is presented.

The ranking of welding robots when the values of both β and λ are chosen as 0.5 is complete. Subsequently, different scenarios were executed. The ranking of welding robots when choosing the value of β as numbers from 0.1 to 0.9 was obtained, as shown in Fig. 1. In the next scenario, the investigation focused on the impact of changing the values of λ, while keeping β constant at 0.5. The rankings of welding robots for different values of λ (ranging from 0.1 to 0.9) are presented in Fig. 2. The ranking results of welding robot types using four methods, MARCOS, PSI, TOPSIS, and RAM, are also displayed in the charts in both figures (Table VI).

TABLE VI. VALUES OF L_i, A_i, R_i, AND RANKINGS OF WELDING ROBOTS

Welding Robot	L_i	A_i	R_i	Rank
WR1	0.03955	0.04500	0.41101	7
WR2	0.04402	0.05777	0.45016	6
WR3	0.05707	0.11667	0.58046	4
WR4	0.06862	0.19251	0.70071	3
WR5	0.09284	0.16542	0.71141	2
WR6	0.08688	0.23125	0.77563	1
WR7	0.07142	0.08236	0.55423	5

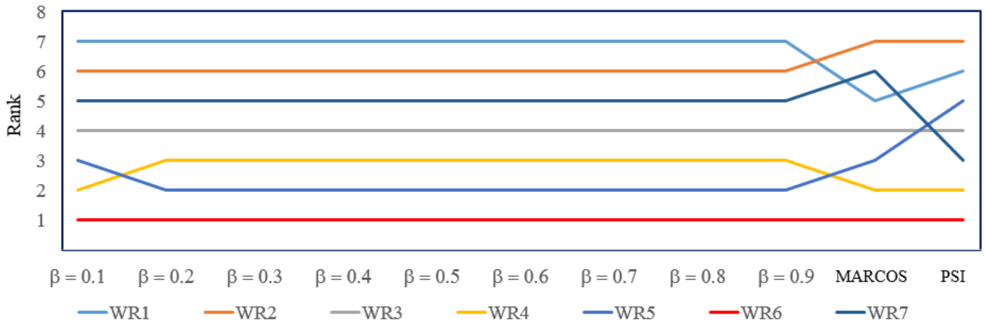

Fig. 1. Ranking of welding robots for different values of β (λ = 0.5).

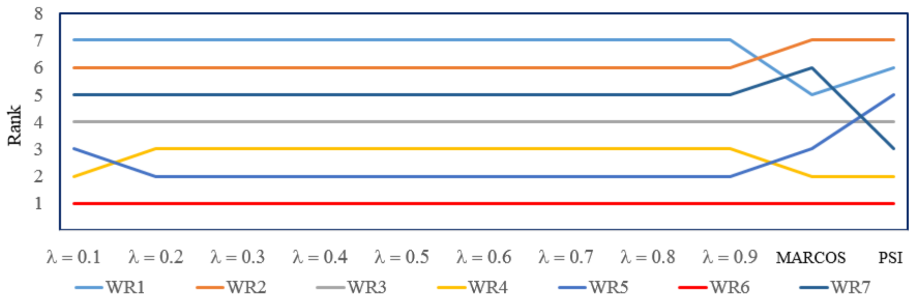

Fig. 2. Ranking of welding robots for different values of λ (β = 0.5).

Figs. 1 and 2 shows that when using the AROMAN method to rank options, their rankings change very little when we vary the values of the two coefficients β and λ. Although β has changed up to ten times, there is only a single exchange of the ranking between option 2 and option 3 when β changes from 0.1 to 0.2. A similar situation occurs when the value of λ changes nine times. This indicates that the two coefficients, β and λ, have little influence on the rankings of the options. Furthermore, in all scenarios, a consistently best option is always identified. Options ranked 4, 5, 6, and 7 are also always the same in all scenarios. Furthermore, the best option found using the *AROMAN* method is always consistent with the best option found using the four methods MARCOS, PSI, TOPSIS, and RAM. All these findings suggest that user coefficients (β and λ) have minimal impact on the best option. The analysis of sensitivity when ranking options in different scenarios is necessary and should be conducted [18, 19]. Scenarios here are understood as changes in MCDM methods used or changes in the coefficients β and λ in the AROMAN method. The Spearman ranking correlation coefficient has been utilized to analyze sensitivity [20]. This coefficient is calculated using Eq. (8) [21].

$$S = 1 - \frac{6\sum_{i=1}^{m} D_i^2}{m(m^2-1)} \tag{8}$$

where *Di* represents the rank difference of the alternatives of a certain scenario compared to another scenario, *m* is the number of alternatives to be ranked.

Each change in the value of β or λ is referred to as a scenario when using the AROMAN method. Table VII summarizes the values of the *S* coefficient in thirteen different scenarios. The first nine scenarios (S1 to S9) correspond to nine values of β. Scenarios S10, S11, S12, and S13 correspond to cases using the MARCOS, PSI, TOPSIS, and RAM methods.

TABLE VII. VALUES OF THE SPEARMAN RANKING CORRELATION COEFFICIENT WHEN CHANGING THE VALUE OF Β AND USING OTHER MCDM METHODS

Si	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13
S1	1	1	0.964	0.964	0.964	0.964	0.964	0.964	0.964	0.964	0.893	0.821	0.857
S2		1	0.964	0.964	0.964	0.964	0.964	0.964	0.964	0.964	0.893	0.821	0.857
S3			1	1	1	1	1	1	1	1	0.857	0.714	0.821
S4				1	1	1	1	1	1	1	0.857	0.714	0.821
S5					1	1	1	1	1	1	0.857	0.714	0.821
S6						1	1	1	1	1	0.857	0.714	0.821
S7							1	1	1	1	0.857	0.714	0.821
S8								1	1	1	0.857	0.714	0.821
S9									1	1	0.857	1	0.821
S10										1	0.857	0.714	0.821
S11											1	0.75	0.964
S12												1	0.607
S13													1

In Table VII, the smallest value of the Spearman ranking correlation coefficient is 0.714, corresponding to the ranking comparison results between scenario S12 and scenarios S3, S4, S5, S6, S7, and S8. This is a relatively high value when evaluating the Spearman ranking correlation coefficient [21]. This confirms the high reliability of the ranking results [21]. Considering only ten different scenarios when changing the value of β, we see that the smallest Spearman ranking correlation coefficient is 0.964, and there are many cases where this coefficient is equal to 1. This further solidifies that changing the value of β has very little impact on the ranking of options when ranked using the AROMAN method.

Table VIII also summarizes the values of the *S* coefficient in thirteen different scenarios. The first nine scenarios correspond to ten values of λ. Scenarios S10, S11, S12, and S13 correspond to cases using the MARCOS, PSI, TOPSIS, and RAM methods.

TABLE VIII. VALUES OF THE SPEARMAN RANKING CORRELATION COEFFICIENT WHEN CHANGING THE VALUE OF Λ AND USING OTHER MCDM METHODS

Si	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13
S1	1	1	0.964	0.964	0.964	0.964	0.964	0.964	0.964	0.964	0.893	0.821	0.857
S2		1	0.964	0.964	0.964	0.964	0.964	0.964	0.964	0.964	0.893	0.821	0.857
S3			1	1	1	1	1	1	1	1	0.857	0.714	0.821
S4				1	1	1	1	1	1	1	0.857	0.714	0.821
S5					1	1	1	1	1	1	0.857	0.714	0.821
S6						1	1	1	1	1	0.857	0.714	0.821
S7							1	1	1	1	0.857	0.714	0.821
S8								1	1	1	0.857	0.714	0.821
S9									1	1	0.857	1	0.821
S10										1	0.857	0.714	0.821
S11											1	0.75	0.964
S12												1	0.607
S13													1

Observing Table VIII, we also notice that the smallest value of the Spearman ranking correlation coefficient is 0.714, corresponding to the ranking comparison results between scenario S12 and scenarios S3, S4, S5, S6, S7, S8, and S10. This result also confirms the high reliability of the ranking results. If we only consider nine different scenarios when changing the value of λ, we find that the smallest λ is 0.964, and there are also many cases where this coefficient is equal to 1. This further strengthens the fact that changing the value of λ has very little impact on the ranking of options when ranked using the AROMAN method.

B. Case 2

Seven types of cutting oils, denoted as CF1, CF2, CF3, CF4, CF5, CF6, and CF7, have been presented for ranking. Each option is characterized by six criteria, including KV -dynamic viscosity, MV-minimum viscosity value, MT - minimum flash point temperature, FP-pour point, PH-pH when diluted at a 5% concentration, and P-price. Among these, KV, MV, MT, and FP are four criteria of type *B*, while PH and P are two criteria of type C. Table VII synthesizes data on various types of cutting oils [22]. The ranking of these cutting oils in this table using the Collaborative Unbiased Rank List Integratio (CURLI) and Proximity Indexed Value (PIV) methods has also been previously conducted [22]. Additionally, TOPSIS and RAM methods have also been used in this case.

The determination of the rankings of cutting oils using the AROMAN method has also been carried out similarly to the approach employed in case 1. In Figs. 3 and 4, respectively, the rankings of cutting oil types when ranked by the AROMAN method in two cases with varying values of β and λ are illustrated. The ranking results of cutting oil types using four methods PIV, CURLI, TOPSIS, and RAM are also displayed in the charts in both figures.

TABLE IX. TYPES OF CUTTING OILS [22]

Cutting Fuild	KV	MV	MT	FP	PH	P
CF1	40	90	150	6	9.25	2.84
CF2	38	85	160	8	11.22	2.72
CF3	38	90	160	8	9.36	2.96
CF4	42	92	200	6	8.52	3.02
CF5	40	96	210	5	7.42	3.22
CF6	38	75	180	6	7.26	3.12
CF7	39	88	190	6	7.26	3.16
Weight	1/6	1/6	1/6	1/6	1/6	1/6

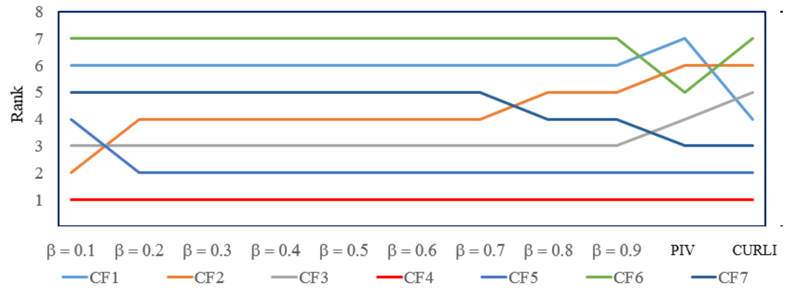

Fig. 3. Ranking of cutting oil types with varying values of β (λ = 0.5).

Observing Fig. 3 reveals that when changing the value of β from 0.1 to 0.2, there is only a permutation between ranking options 2 and 4. Similarly, when changing β from 0.7 to 0.8, there is only a permutation between options 4 and 5. In all other scenarios, the rankings of options remain consistent. This indicates that the value of β has minimal influence on the rankings of options. Especially, in all conducted scenarios (when changing β), the best option found using the AROMAN method is also consistently the best option found using the four methods PIV, CURLI, TOPSIS, and RAM.

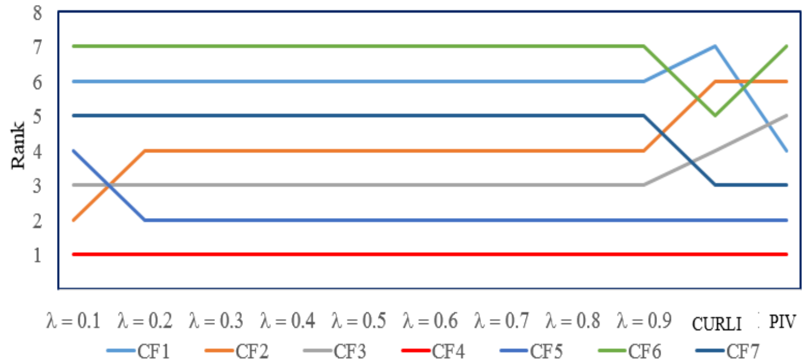

Fig. 4. Ranking of cutting oil types with varying values of λ (β = 0.5).

According to the observations in Fig. 4, when using the *AROMAN* method, there is only a single permutation in the ranking of options 2 and 4 when λ takes two different values, namely 0.1 and 0.2. In all other scenarios, the rankings of options are entirely consistent. A notable point reiterated is that the top-ranked option found using the *AROMAN* method (in 9 scenarios when changing λ) is always consistent and is also the best option found using the four methods CURLI, PIV, TOPSIS, and RAM. All discussions indicate that the value of λ has minimal impact on the rankings of options.

The calculation of the Spearman ranking correlation coefficient is also carried out in this case. In Table X, the synthesis of the *S* coefficients for thirteen scenarios is presented, where the first nine scenarios correspond to ten values of β, and the remaining four scenarios correspond to the cases of using the PIV, CURLI, TOPSIS, and RAM methods. In Table XI, the synthesis of the *S* coefficients for thirtteen scenarios is provided, where the first nine scenarios correspond to nine values of λ, and the remaining four scenarios correspond to the cases of using the PIV, CURLI, TOPSIS, and RAM methods.

TABLE X. VALUES OF THE SPEARMAN RANKING CORRELATION COEFFICIENT WHEN CHANGING THE VALUE OF β AND USING OTHER MCDM METHODS

Si	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13
S1	1	0.857	0.857	0.857	0.857	0.857	0.857	0.75	0.75	0.464	0.429	0.464	0.964
S2		1	1	1	1	1	1	0.964	0.964	0.75	0.714	0.607	0.893
S3			1	1	1	1	1	0.964	0.964	0.75	0.714	0.607	0.893
S4				1	1	1	1	0.964	0.964	0.75	0.714	0.607	0.893
S5					1	1	1	0.964	0.964	0.75	0.714	0.607	0.893
S6						1	1	0.964	0.964	0.75	0.714	0.607	0.893
S7							1	0.964	0.964	0.75	0.714	0.607	0.893
S8								1	1	0.857	0.821	0.75	0.821
S9									1	0.857	0.821	1	0.821
S10										1	0.75	0.893	0.536
S11											1	0.607	0.464
S12												1	0.571
S13													1

TABLE XI. VALUES OF THE SPEARMAN RANKING CORRELATION COEFFICIENT WHEN CHANGING THE VALUE OF λ AND USING OTHER MCDM METHODS

Si	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13
S1	1	0.857	0.857	0.857	0.857	0.857	0.857	0.857	0.857	0.464	0.429	0.464	0.964
S2		1	1	1	1	1	1	1	1	0.75	0.714	0.607	0.893
S3			1	1	1	1	1	1	1	0.75	0.714	0.607	0.893
S4				1	1	1	1	1	1	0.75	0.714	0.607	0.893
S5					1	1	1	1	1	0.75	0.714	0.607	0.893
S6						1	1	1	1	0.75	0.714	0.607	0.893
S7							1	1	1	0.75	0.714	0.607	0.893
S8								1	1	0.75	0.714	0.607	0.893
S9									1	0.75	0.714	1	0.893
S10										1	0.75	0.893	0.536
S11											1	0.607	0.464
S12												1	0.571
S13													1

Observing the data in Tables X and XI, we also note that when changing the value of β, λ by a factor of nine (from 0.1 to 0.9), the Spearman correlation coefficient ranks for these scenarios still have very large values, with the smallest being 0.75, and in many cases, this coefficient equals 1. This once again affirms that changing the values of β and λ has minimal impact on the rankings of alternatives.

Through the examination of two examples from different domains (robotic welding ranking and cutting oil ranking), it is evident that user coefficients (β and λ) have minimal impact on the rankings of options. The variation in the values of these coefficients does not affect the identification of the best option using the AROMAN method, and the best option consistently aligns with the best option identified using other MCDM methods.

IV. CONCLUSION

In investigating the influence of the two user coefficients (β and λ) on the rankings of alternatives using the *AROMAN* method, important results have been obtained:

Firstly, altering the values of the coefficients β and λ has very little impact on the rankings of alternatives. This demonstrates the stability and reliability of the AROMAN method, providing favorable conditions for users when applying it in multi-criteria decision-making.

Secondly, the most crucial point is that the best alternative, when utilizing the AROMAN method, is not dependent on the specific values of the coefficients β and λ. In fact, this result indicates that the best alternative is always equivalent to the best alternative when using other MCDM methods. This brings about flexibility and the independent nature of AROMAN, instilling confidence in users when selecting and applying this method.

Considering these discoveries, we believe that users can be assured when deciding to use the AROMAN method in multi-criteria decision-making. Furthermore, the lack of dependence on specific values of β and λ opens up new prospects for the widespread application of this method in practical scenarios.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

Hoang Xuan Thinh initiated the idea and wrote the first version of the article. Tran Van Dua provided feedback. Both collaborated to restructure the article's content. All authors had approved the final version.

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