# Research on the Mechanism of Parallel Structure with Circular Guide 

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#### Abstract

A new structural diagram of a parallel structure mechanism with a circular guide has been developed, which consists of hybrid and cross-coupling kinematic chains. Hybridity lies in the combination of parallel and sequential kinematic chains., additionally, the block diagram of the mechanism contains cross-coupling chains between parallel circuits. Compared to known analogs, the mechanism under consideration utilizes an original hinge design, which increases the working area. An algorithm has been developed to solve the problem of the position of the output link of the mechanism and another algorithm to determine the working area. A numerical example of solving the inverse kinematics problem is given.


Keywords-hybrid parallel structure mechanism, circular guide, cross connections, cross-constraint, structural analysis, inverse kinematics problem, working area

## I. Introduction

Currently, spatial mechanisms utilizing a circular guide have found practical application application applications in various fields of engineering and technology in positioning tasks [1], laparoscopic surgery [2], motion simulation devices [3], in simulators [4, 5]. In the work [6], The authors analyze various problems associated with the use of hybrid manipulators in additive technologies, medical robots, and manipulators for aggressive environments. It should be noted that mechanisms of a parallel structure, including mechanisms with a circular guide - rotopods, have a limited working area. This is due to the presence of a large number of kinematic chains, links, as well as kinematic pairs that have restrictions on the mutual relative movement of links in movable joints.

Mechanisms with a circular guide, in their functional properties, complement the well-known class of GoughStewart mechanisms [7-10] with the possibility of full rotation around a vertical axis.

A design feature of mechanisms with a circular guide is the installation of drives closer to the base, directly on the

[^0]carriages, to reduce the inertial properties of the mechanism. This allows the output link of the mechanism to move at high speeds and work with heavy loads, which significantly expands the functionality of these mechanisms. Mechanisms with parallel crisscrosscoupling coupling cross-coupling chains are also known [11], which can have reduced dimensions of the base and output link and increased rigidity of the mechanism. Depending on the number of kinematic chains, six-moving mechanisms with the number of kinematic chains from three to six are distinguished. For example, mechanisms with three kinematic chains are known, with the structure 3-RRPS [12] and 3-RPRS [13, 14], based on which mechanisms for simulators have been developed [15].

Mechanisms with the structure 3-RPPRS, 3-PPRS, and 3-RRPRS and with three kinematic chains have been developed, in which, to increase the size of the working area, one additional drive was introduced into each of the chains $[16,17]$.

The article presents a new spatial mechanism [18] with a hybrid structure consisting simultaneously of parallel and sequential kinematic chains, in which the drives are located directly on the carriages and cross chains move along the circular guides of the base. The mechanism uses an original hinge design - a mini platform, in the form of a three-moving joint, in which rotations around three mutually orthogonal axes are much greater than in a spherical hinge. The presence of a hybrid kinematic chain allows one to significantly expand the boundaries of the working area of the mechanism [8].

The purpose of this study is to synthesize a new structural diagram of a mechanism with an increased working area, which is achieved by installing a miniplatform in each kinematic chain containing a link that sequentially connects the mini-platform with the output link of the mechanism. In this case, there is no need to install an additional engine to increase the size of the working area.

An algorithm is proposed for determining the boundaries of the working area for the proposed new mechanism.

## II. Designing of the Parallel Structure Mechanism Under Study

To carry out the research, a 3D model of the mechanism under study was created (Fig. 1), in which the output link 1 (moving platform) is connected to the base 2 (circular guide) using three identical kinematic chains. Each of the three structurally identical kinematic chains consists of parallel-cross and series sub-chains.


Fig. 1. Spatial parallel structure mechanism with a circular guide.
The kinematic chains of the mechanism (Fig. 1) consist of a parallel subchain consisting of a drive carriage 3 and a non-drive carriage 4 , which move along a circular guide 2 and are connected to the upper intermediate mini platform 5 using two parallel kinematic subchains made in the form of rods 6 and 7 . Rods 6 and 7 are connected at one end to carriages 3 and 4 via a spherical hinge, and at the other end to mini platform 5 via a rotary hinge, the axes of which are located coaxially.

The movement to output link 1 is transmitted by a sequential circuit in the form of a " T "-shaped link 8 with rotational hinges, the axes of which are mutually orthogonal. Thus, on mini platform 5 there is an intersection of three mutually orthogonal axes, which is an analog of a spherical hinge. It should be noted that in the previously known classical structural diagrams of rotopods [8, 18], the kinematic chains are connected to the movable platform using spherical hinges. The mechanism under consideration has a hinge unit, which is an analog of a spherical joint but has improved kinematic characteristics.

The technical specifications for spherical joints indicate a service angle not exceeding $20^{\circ}$. In the proposed design of the mini-platform hinge unit (Fig. 2), rotations around the $X_{A}$ and $Z_{A}$ axes of the rotational pairs are not limited, and the service angle around the $Y_{A}$ axis can reach $50^{\circ}$, which significantly increases the range of rotation angles around mutually orthogonal axes.


Fig. 2. Mini platform mechanism.
In addition, each of the three kinematic chains contains a cross RPR subchain, consisting of links 9 and 10 (Fig. 3) which are connected by a translational ( P ) pair in the form of a linear motor, and this subchain is connected to carriages 3 and 4 by rotational hinges (B), whose axes are perpendicular to the plane formed by the circular guide 2 .


Fig. 3. Kinematic chain RPR.
Thus, in the proposed mechanism, the movable joints have an increased service angle.

## III. Structure Analysis of the Mechanism

The number of degrees of freedom of the mechanism is determined by the classical Somov-Malyshev formula [19].

$$
\begin{equation*}
W=6 \times n-\sum_{i=1}^{5} \dot{\times} \times p_{i} \tag{1}
\end{equation*}
$$

where n is the number of moving links, $\mathrm{P}_{\mathrm{i}}$ is the number of one, two, three, four, and five moving kinematic pairs in the mechanism, $i=1, \ldots, 5$.

When structurally analyzing the mechanism, it is necessary to consider the presence of RPR subchains (Fig. 2), which are formed by a lower pair of the fifth class and are flat dyads belonging to the Assur group. All three kinematic subchains of the RPR perform the plane-parallel motion, parallel to the plane of the circular guide. It is known that the number of degrees of freedom of such subchains is zero and when they are attached to the mechanism, they do not change the number of degrees of freedom [8, 20].

Consequently, when calculating the number of degrees of freedom, they can be ignored and one can proceed to a replacement structural diagram of the mechanism (Fig. 4)
without RPR circuits. Thus, according to Eq. (1), the number of degrees of freedom will be equal to:

$$
W=6 \times 19-5 \times 18-3 \times 6=6
$$

where $n=19$ is the number of moving links, $P_{5}=18$ is the number of single-moving pairs, $P_{3}=6$ is the number of three-moving pairs.

The structural analysis carried out shows that Eq. (1) is not "sensitive" to the structural features of the mechanism. In this case, to the geometry of a spatial mechanism in which kinematic chains are performing the plane-parallel motion, but in Eq. (1) the links of these chains will be considered as spatial.

Thus, a new spatial hybrid mechanism of a parallel structure with a circular guide and six degrees of freedom was synthesized.


Fig. 4. Mechanism without RPR kinematic chains.

## IV. Solution of the Inverse Problem of Kinematics

Solving the inverse kinematics problem allows one to calculate the generalized coordinates of the mechanism based on the given absolute coordinates of the output link and determine the boundaries of the working area of the mechanism.

In the fixed coordinate system OXYZ (Fig. 4), the absolute coordinates of the output link are specified:

$$
X=\left[x_{o}, y_{o}, z_{o}, \alpha, \beta, \gamma\right]^{T}
$$

where $x_{o}, y_{o}, z_{o}$ are the coordinates of the origin of the moving coordinate system $x_{o}, y_{o}, z_{o}$, and $\alpha, \beta, \gamma$ are the orientation angles of the axes of the moving coordinate system ( $\alpha$ is the angle of rotation around the $X$ axis, $\beta$ is the angle of rotation around the $Y$ axis, $\gamma$ is the angle of rotation around the Z axis).

It is necessary to determine the generalized coordinates $q_{i}(i=1,2 \ldots, 6)$, which are the orientation angles of the drive carriages $\varphi_{1}, \varphi_{2}, \varphi_{3}$ and the distances between the drive and non-drive carriages $l_{1}, l_{2}, l_{3}$ for each kinematic chain of the mechanism:

$$
\begin{aligned}
& q_{1}=\varphi_{1}, q_{4}=l_{1} \\
& q_{2}=\varphi_{2}, q_{5}=l_{2} \\
& q_{3}=\varphi_{3}, q_{6}=l_{3}
\end{aligned}
$$

The drive carriages are located at points $B_{1}, B_{2}$, and $B_{3}$ and move along a fixed circular guide (Fig. 5). Fig. 5 shows only one kinematic chain, the other two chains are similar. The non-drive carriages are located at points $B_{1}^{\prime}$, the position of which is determined by the chords $B_{1} B_{1}^{\prime}$, the lengths of which $l_{1}$, are determined by linear drives. As an example, we will determine the generalized coordinates of the mechanism for one kinematic chain given the absolute coordinates of the output link.


Fig. 5. Solving the position problem for one kinematic chain.
In the kinematic chain under consideration, the axes of the rotational pairs on the mini platform intersect at point $\mathrm{A}_{1}$, which can be considered as the center of the spherical hinge, the coordinates of which need to be determined.

Let us denote the coordinates of the platform point $A_{1}$ in the fixed coordinate system of the base $P_{\mathrm{A} 1}=$ $\left[P_{\mathrm{A} 1}^{x}, P_{\mathrm{A} 1}^{y}, P_{\mathrm{A} 1}^{z}\right]^{T}$, which is determined from the vector Eq. (2).

$$
\begin{equation*}
P_{\mathrm{A} 1}=\rho+R \times P_{p A 1} \tag{2}
\end{equation*}
$$

where the vector $\left[x_{o}, \mathrm{y}_{o}, \mathrm{z}_{o}\right]^{T}$ determines the position of the beginning of the moving coordinate system of the output link relative to the fixed OXYZ coordinate system of the base, and the vector $P_{\mathrm{pA} 1}=\left[x_{\mathrm{pA} 1}, y_{\mathrm{pA} 1}, z_{\mathrm{pA} 1}\right]^{T}$ determines the position of point $\mathrm{A}_{1}$ of the output link in the moving coordinate system of the platfom $x_{o}, \mathrm{y}_{o}, \mathrm{z}_{o}$. Three angles $\alpha, \beta, \gamma$ determine the angular orientation of the output link. The rotation matrix R of the moving system relative to the stationary system is found by the formula: $R=R_{z}(\gamma) \times R_{y}(\beta) \times R_{x}(\alpha)$
where $R_{z}(\gamma), R_{y}(\beta), R_{x}(\alpha)$ three rotation matrices

$$
R_{z}(\gamma)=\left[\begin{array}{l}
\cos \gamma-\sin \gamma 0 \\
\sin \gamma \cos \gamma 0 \\
001
\end{array}\right], R_{y}(\beta)=\left[\begin{array}{l}
\cos \beta 0 \sin \beta \\
010 \\
-\sin \beta 0 \cos \beta
\end{array}\right]
$$

$$
R_{x}(\alpha)=\left[\begin{array}{l}
100 \\
0 \cos \alpha-\sin \alpha \\
0 \sin \alpha \cos \alpha
\end{array}\right]
$$

$\mathrm{R}=\left[\begin{array}{l}\cos \beta \cos \gamma \sin \alpha \sin \beta \cos \gamma-\cos \alpha \sin \gamma \sin \alpha \sin \gamma+\cos \alpha \sin \beta \cos \gamma \\ \cos \beta \sin \gamma \cos \alpha \cos \gamma+\sin \alpha \sin \beta \sin \gamma \cos \alpha \sin \beta \sin \gamma-\sin \alpha \cos \gamma \\ -\sin \beta \sin \alpha \cos \beta \cos \alpha \cos \beta\end{array}\right]$
Let us denote $A_{1} B_{1}=\mathrm{A}_{1} B_{1}^{\prime}=\mathrm{L}$, where is a structurally specified value, and $O B_{1}=0 B_{1}^{\prime}=r$, where r is the radius of the circular guide of the mechanism.

To determine the position of point $\mathrm{B}_{1}$, we create a system of vector Eq. (3).

$$
\left\{\begin{array}{l}
\left(P_{A 1}-P_{B 1}\right)^{2}=L^{2}  \tag{3}\\
P_{B 1}=\left[\begin{array}{l}
\mathrm{r} \cos \varphi \\
\mathrm{r} \sin \varphi \\
0
\end{array}\right]
\end{array}\right.
$$

where $P_{A 1}$ and $P_{B 1}$ are radius vectors of points $\mathrm{A}_{1}$ and $\mathrm{B}_{1}$, respectively, in a fixed coordinate system. The geometric meaning of relations Eq. (3) is to find the coordinates of the intersection points of two geometric figures: a sphere of radius L and a circle of radius $r$.

In Eq. (3):

$$
\begin{equation*}
P_{A 1}^{2}-2 P_{A 1} \times P_{B 1}+P_{B 1}^{2}=L^{2} \tag{3}
\end{equation*}
$$

Taking into account the coordinates of the vectors $P_{A 1}$ in Eq. (2) and $P_{B 1}$, we obtain the equation for the relatively unknown $\varphi$ :

$$
\begin{equation*}
P_{\mathrm{A} 1}^{x} \times \cos \varphi+\mathrm{P}_{\mathrm{A} 1}^{y} \sin \varphi=\mathrm{k} \tag{4}
\end{equation*}
$$

where $k=\frac{P_{A 1}^{2}+r^{2}-L^{2}}{2 \times r}$ is a constant for a given position of the output link.

To find $\varphi$ of their Eq. (4), we use the well-known trigonometric relation::

$$
\left\{\begin{array}{l}
\sin \varphi=\frac{2 \times t}{1+t^{2}} \\
\cos \varphi=\frac{1-t^{2}}{1+t^{2}}
\end{array}\right.
$$

where $t=\tan \frac{\varphi}{2}$ then from Eq. (4) we get:

$$
P_{\mathrm{A} 1}^{x} \times \frac{1-t^{2}}{1+t^{2}}+P_{\mathrm{A} 1}^{y} \times \frac{2 \times t}{1+t^{2}}=k
$$

As a result, we have a quadratic equation for the variable $t$ :

$$
\begin{equation*}
\left(k+P_{A 1}^{x}\right) \times t^{2}-2 \times P_{A 1}^{y} \times t+\left(k-P_{A 1}^{x}\right)=0 \tag{5}
\end{equation*}
$$

Eq. (5) is a general quadratic equation:

$$
a \times t^{2}+b \times t+c=0
$$

Whose roots look like:

$$
\begin{equation*}
t_{\mathrm{i} 1, \mathrm{i} 2}=\frac{-\mathrm{b} \pm \sqrt{b^{2}-4 \times \mathrm{a} \times \mathrm{c}}}{2 \times \mathrm{a}} \tag{6}
\end{equation*}
$$

where $a=k+P_{A 1}^{x}, b=-2 \times P_{A 1}^{y}, c=k-P_{A 1}^{x}, \mathrm{i}=1$ is the number of the kinematic chain.

The condition for the existence of the mechanism is described by the following inequalities:
$a=k+P_{A 1}^{x} \neq 0, b^{2}-4 \times a \times c>0$
The first inequality excludes division by zero in Eq. (6), and the second excludes the absence of complex roots in Eq. (6)

The double sign $\pm$ in front of the radical in Eq. (6) indicates the presence of two solutions to the variable $t$, which correspond to the positions of two carriages $B_{1}$ and $B_{1}^{\prime}$ on the circular guide.

Thus, we get:

$$
\begin{equation*}
\varphi_{11,12}=2 \times \operatorname{arctg}\left(t_{11,12}\right) \tag{7}
\end{equation*}
$$

Next, we define a linear generalized coordinate:

$$
\begin{equation*}
l_{1}=r \times \sqrt{2 \times(1-\cos \theta)} \tag{8}
\end{equation*}
$$

where $\theta=\varphi_{12}-\varphi_{11}-2 \times \delta$ and the angle $\delta$ is a structurally specified parameter (Fig. 4).

From Eqs. (7) and (8) we can determine the required generalized coordinates $q_{1}=\varphi_{11}, q_{4}=l_{1}$. To determine the remaining generalized coordinates of the mechanism, it is necessary to write down the system of Eq. (3) for two other kinematic chains.

Thus, based on the geometry of the mechanism, equations were obtained that connect the absolute coordinates of the output link and the generalized coordinates of input link, and an algorithm for their calculation was developed

## V. An Example of Solving The Problem of Determining the Working Area

Works consider various aspects of solving problems of determining the boundaries of the work area [21-25]. In Ref. [21], the boundaries of the working area are established by the condition that the projection of the center of mass of the output link is located inside the circular guide. In other words, the criterion for the boundaries of the working area is the special positions of the mechanism.

Based on the above mathematical model for solving the inverse problem of positions, an algorithm for determining the boundaries of the working area was developed, and a program was implemented in the MATLAB environment to determine the working area of a spatial six-moving mechanism with a circular guide with three kinematic chains.

To solve the problem, the geometric parameters of the mechanism and design restrictions in kinematic pairs are specified, with known limiting values of the generalized coordinates $q_{i}(i=1,2 \ldots, 6)$. The problem is solved by the method of discretizing space by points of an initially specified grid, as well as sampling parameters and scanning area, followed by checking the specified restrictions. For each grid point, an analysis of permissible displacements in all kinematic pairs of the mechanism is carried out. The calculation of the working area is carried out both for fixed orientation angles of the output link and
for their specified ranges. Next comes the processing of the resulting data array of coordinates of points in the working area of the mechanism and visualization of the calculation results

Initial data are shown in Table I.
TABLE I. SIzE of PARAMETERS

| Serial | Parameters | Size |
| :---: | :---: | :---: |
| 1 | The radius of the circular guide, $r(\mathrm{~mm})$ | 60 |
| 2 | Platform radius (output link), $(\mathrm{mm})$ | 40 |
| 3 | Rod length, $\mathrm{L}(\mathrm{mm})$ | 100 |
| 4 | The minimum length of the cross-chain is $\mathrm{l}_{1}(\mathrm{~mm})$ | 30 |
| 5 | The maximum length of the cross-chain is $1_{1}(\mathrm{~mm})$ | 100 |
| 6 | Output link orientation angles: $\alpha, \beta, \gamma$ | $0^{\circ}$ |
| 7 | Permissible angle of approach of carriages | $10^{\circ}$ |

The scanning area was set in the form of a cylinder along the $Z$ axis with a radius of 10 mm (Fig. 4), inside which a parallelepiped with dimensions is inscribed: -10 $\mathrm{mm} \leq x \leq 10 \mathrm{~mm} ;-10 \mathrm{~mm} \leq y \leq 10 \mathrm{~mm} ;-80 \mathrm{~mm} \leq z \leq 100$ mm . Sampling step $\Delta x=\Delta y=\Delta z=1 \mathrm{~mm}$.

Based on the initial data (Table I), the working area of the mechanism was calculated (Fig. 6).


Fig. 6. Working area of the mechanism.
To solve the inverse problem of position, functions for changing the absolute coordinates of the output link were specified at a constant orientation of the output link, in the form of movement in an ascending spiral (Fig. 7):


Fig. 7. The trajectory of the output link according to a given law.
$x=5 \sin (\mathrm{t})(\mathrm{mm}), y=5 \cos (\mathrm{t})(\mathrm{mm}), z=95-5 \mathrm{t} / 2 \pi(\mathrm{~mm})$, $\alpha=\beta=\gamma=0$, where the movement time varied within $0 \leq$ $t \leq 5 \mathrm{~s}$. According to to to the given law of motion of the output link, the functions of changing generalized coordinates $q_{1}, q_{2}, \ldots, q_{6}$ were determined using Eqs. (7) and (8).

Fig. 8 shows diagrams of changes in the generalized angular $q_{1}, q_{2}, q_{3}$ and linear $q_{4}, q_{5}, q_{6}$ coordinates of the mechanism.


Fig. 8. Graph of changes (a) in angular coordinates (b) in linear coordinates.

## VI. Conclusion

The article synthesizes a diagram of a new parallel structure mechanism with a circular guide, which has expanded functional properties, namely an increased working area, without introducing additional motors by increasing the length of the series chain. This reduces the inertial properties of the mechanism and simplifies the control system.

A structural analysis of a new hybrid parallel structure mechanism with a circular guide is carried out. The structural features of the mechanism were analyzed, which made it possible to determine the number of degrees of freedom. Structural analysis shows that different kinematic chains belonging to Assur groups can be chosen as flat dyads, which makes it possible to synthesize a line
of new mechanisms. The kinematic properties of the new mechanism were simulated.

An algorithm for solving the inverse kinematics problem has been developed and a numerical example has been successfully solved.

Based on the results of calculations using an algorithm for solving the inverse kinematics problem, it was established that the mechanism is capable of providing the specified law of output motion.

## Conflict of Interest

The authors declare no conflict of interest.

## Author Contributions

Dinh Tung Vo, Rashoyan G. V, Aleshin A. K, Kondratyev I. M conducted the research and wrote the paper. Sergey Kheylo analyzed the data and checked the paper. All authors had approved the final version.

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