A Novel Model Predictive Control for an Autonomous Four-Wheel Independent Vehicle

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Abstract—**This work is centered on developing a novel Model Predictive Control (MPC) for Four-Wheel Independent (FWID) vehicles to achieve trajectory tracking effectiveness, which is difficult to satisfy due to the changing the optimization solution after each time period. By using linearization technique for FWID model and eliminating the term of dynamic uncertainty in the tracking error model, the nominal linear Discrete Time System (DTS) is achieved to develop the proposed MPC strategy, which leverages the Luenberger observer to obtain the predictive model and extends for obtaining the Output feedback MPC scheme. On the other hand, the appropriate optimization problem is given at each time instant to guarantee the stability of the closed loop system under the designed MPC law without the consideration of terminal region as well as terminal controller, which have been considered in the previous researches. The unification between the optimization problem in MPC scheme and the tracking problem is validated by the Lyapunov function-based analysis with the inequality estimations. The efficiency of the proposed MPC law for FWID vehicles is clarified through a simulation study.**

Keywords—**four-wheel independent vehicles, discrete time system, model predictive control, trajectory tracking control**

I. Introduction

The Four-Wheel Independent (FWID) vehicles have garnered considerable attention in the recent years for the trajectory tracking requirement [1–3]. Moreover, the control objective is also expanded for optimal control problem with the advantage of handling state, input signals constraint. The development of MPC strategy can be considered as an appropriate approach for FWID vehicles. The control objective is to guarantee the unification of trajectory tracking requirement and optimal control problem. Some traditional nonlinear control schemes were presented in [1, 2]. The Linear Matrix Inequality (LMI)-based Sliding Mode Control (SMC) law was developed after obtaining the Multiple-Input Multiple-Output (MIMO) linear tracking error system [1]. However, it requires to obtain the Fuzzy laws in this work and the disadvantage of a chattering phenomenon [1]. Moreover, the SMC approach was integrated fault diagnosis technique for FWID vehicles to improve the control performance [2]. Nevertheless, the consideration of Proportional-Integral (PI) controller in a channel was difficult to guarantee the tracking problem with time-varying [2]. Additionally, the SMC approach was also investigated in different vehicles, such as nonholonomic Wheeled Mobile Robots (WMRs) [3, 4]. A disturbance based nonsingular Recursive-Structure Sliding Model (RSSM) control law was developed for a general high-order nonlinear system and the kinematic sub-system of WMRs was considered a particular case [3]. Additionally, the kinematic sub-system of WMRs can be designed using only Cartesian position and velocity without the knowledge of attitude and acceleration [4]. The proposed method was investigated by using the transforming model and trigonometry formulas [4]. Although the classical nonlinear control methods had attracted the increasing attention, it is still limited in engineering application owing to the optimal control problem.

To overcome the difficulty of developing optimal control problem in robotic systems, an actor/critic Reinforcement Learning (RL) method was putted forward in [5, 6]. The idea is to train both networks of actor and critic to handle the approximating solution of Hamilton Jacobi Bellman (HJB) equation, which is impossible to directly solve. Furthermore, the conventional optimal control problem can be improved by Model Predictive Control (MPC) schemes (see Table I), which is one of the most efficient control approaches for handling physical constraints [7–21]. The technical progresses of developing MPC mainly lie in two aspects. The first aspect was to apply MPC schemes for linear systems [5–7, 17, 19, 21], and the references therein. Initially, the researches chiefly concentrate on linear Discrete-Time systems for set-point tracking MPC with avoidance features to be ensured that the evolution of the output signal lies outside any non-feasible output region [9]. For the stability satisfaction of MPC algorithm, the optimization problem was established at each time instant with the constraint set to be achieved

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from LMI, which contains the information of linear model, external disturbance, controller and the quadratic form of upper bound [17]. Moreover, the implementation of MPC nonlinear robotic systems was indirectly considered via the appropriate linear systems [5, 6, 19, 21]. The safety-guaranteed ship berthing was proposed after decomposing into surge and sway-yaw subsystems, which are linearized for developing MPC algorithm [7]. By implementing the linearization technique, two event-triggered MPC laws were presented for the corresponding linear model with the constraints of inputs, states being given [8]. Based on Taylor series linearization, the MPC algorithm was developed for outer control loop of underactuated Autonomous Underwater Vehicles (AUVs) [19]. Besides, the Extend State Observer (ESO) based nonlinear control scheme was developed for the remaining inner control loop [19]. By choosing the suitable control input in Omni robots, LMI based MPC strategy was presented for a polyhedral LPV Discrete Time System (DTS) [21]. Additionally, the connection between RL algorithm and MPC law was describe to point out the satisfaction of the convergence performance in not only optimal control solution but also the tracking error [13]. On the other hand, the second aspect was to directly apply MPC algorithms for non-linear systems [10–12, 14, 16, 18, 20, 21], and the references therein. The Recurrent Neural Networks (RNNs) were used to describe a nonlinear system as a Gated-Recurrent Unit (GRU), which was controlled by observer based MPC laws [11, 12]. However, the stability consideration of above MPC laws requires the achievement of the appropriate constraint set as well as the challenge of choosing the terminal controller and terminal region.

TABLE I. THE COMPARISON TABLE OF METHODS

References	MPC for Linear Systems	MPC for Nonlinear Systems	Stability consideration with choosing the terminal controller and terminal region
[7–9, 15, 17, 19]	X		X
[8–10, 12, 16, 18, 20, 21]		X	X

Motivated by the above MPC laws and FWID control systems, this article focus on developing a novel MPC law for a FWID to guarantee the trajectory tracking control problem. The primary contributions are introduced in the following:

(1) One is that unlike the previous studies of Lyapunov based classical control schemes, such as [1–4], the MPC control framework was proposed for FWID systems with the proposed observer being considered as the predictive model. Moreover, this technique was applied for the approximate discrete-time linear system with the appropriate optimization problem to be established at each time instant.

(2) The other is that the unification of optimization in MPC law and stability is guaranteed. It is important to emphasize that the stability consideration in this article distinguishes from those utilized in [5–12, 15–21].

The organization of the subsequent sections of this article is as follows. In Section II, the preliminaries and problem statements are given. In Section III, we further present the proposed MPC for a FWID with theorems and lemmas. Section IV presents the simulation results to demonstrate their performance. This paper is gathered with a summary of the findings in Section V.

II. PRELIMINARIES AND PROBLEM STATEMENTS

Fig. 1 depicts the configuration of an autonomous four-Wheel Independent Vehicles (4WID) [1].

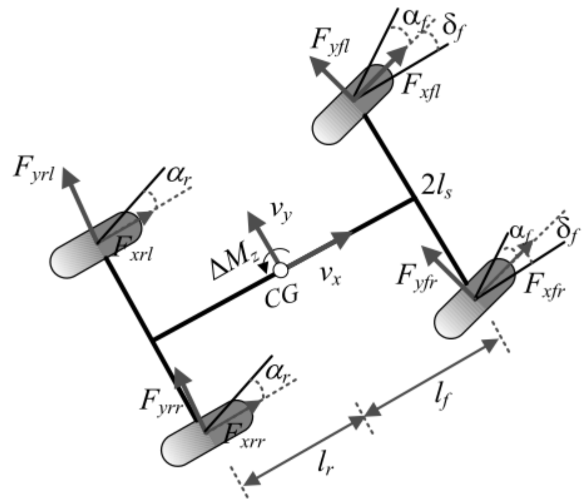

Fig. 1. Dynamic model of autonomous four-Wheel Independent Vehicles (4WID) [1].

For the purpose of obtaining the Model Predictive Control (MPC) design of 4WID, some essential assumptions and lemmas are utilized throughout this article.

Assumption 1: The roll, pitch, vertical motion and the difference of tire cornering descriptions between the left and right wheels are ignored.

Assumption 2: The longitudinal velocity υ_x is considered as a constant value.

In the light of Ref. [1], according to Newton's theorem and above assumptions, a 4WID can be described as the following dynamic equations (Fig. 1) with the parameters and variables to be defined in Table II:

$$\begin{cases} m\upsilon_x\left(\dot{\gamma}+s\right)=F_{yf}+F_{yr} \\ I_z\dot{s}=l_fF_{yf}-l_rF_{yr}+\Delta M_z \end{cases} \tag{1}$$

where $\Delta M_z=\left(F_{xfr}-F_{xfl}\right)l_s\cos\delta_f+2l_f\sin\delta_f+\left(F_{xrr}-F_{xrl}\right)l_s$ (2)

and lateral forces can be formulated as:

$$\begin{cases} F_{yf} = C_f a_f = C_f \left(\delta_f - \dfrac{l_f s}{\upsilon_x} - \gamma \right) \\ F_{yr} = C_r a_r = C_r \left(\dfrac{l_f s}{\upsilon_x} - \gamma \right) \end{cases} \quad (3)$$

TABLE II. THE PHYSICAL MEANINGS OF NOTATIONS

Notation	Meaning
γ, s	The Slip angle, yaw rate, respectively.
I_z	The vehicle inertia with respect to the center of gravity
υ_x, δ_f	The longitudinal speed and the front steering orientation
F_{yi}, F_{xi}	The longitudinal force and the lateral tire force of i th tire ($1 \to fl,\ 2 \to fr,\ 3 \to rl,\ 4 \to rr$)
C_f, C_r	The front, rear cornering stiffness
a_f, a_r	The front, rear tire sideslip angles
m	Mass of the vehicle (Fig. 1)
e_y, e_a	The lateral, angle errors (Fig. 1)
D_l, K_l	The previewed distance, the curvature of the desired trajectory (Fig. 1)
MIMO	Multiple-input Multiple-output

In the view of the trajectory following model (Fig. 2), it follows that its model can be expressed by:

$$\begin{cases} \dfrac{d}{dt} e_y = \upsilon_x \left(e_a - \gamma \right) - s D_l \\ \dfrac{d}{dt} e_a = \upsilon_x K_l(t) - s \end{cases} \quad (4)$$

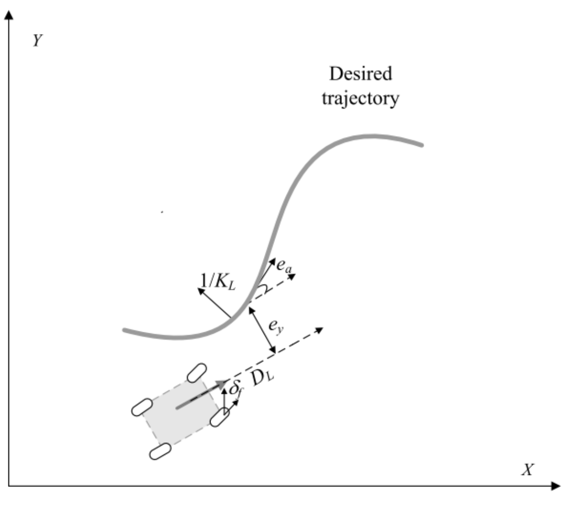

Fig. 2. Trajectory following control [1].

According to Eqs. (1) and (4), we achieve the tracking error model of 4WID as shown in the following MIMO linear system:

$$\begin{cases} \dfrac{d}{dt} x = \left(A + \Delta A \right) x + \left(B + \Delta B \right) u + D K_l \\ y = C x \end{cases} \quad (5)$$

where:

$$A = \begin{bmatrix} 0 & \upsilon_x & -\upsilon_x & -D_l \\ 0 & 0 & 0 & -1 \\ 0 & 0 & \dfrac{-\left(C_r + C_f \right)}{m \upsilon_x} & -\dfrac{\left(l_f C_f - l_r C_r \right)}{m \upsilon_x^2} - 1 \\ 0 & 0 & \dfrac{\left(-l_f C_f + l_r C_r \right)}{I_z \upsilon_x} & \dfrac{-\left(l_f^2 C_f + l_r^2 C_r \right)}{I_z \upsilon_x} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{C_f}{m \upsilon_x} & 0 \\ \dfrac{l_f C_f}{I_z} & \dfrac{1}{I_z} \end{bmatrix}, D = \begin{bmatrix} 0 \\ \upsilon_x \\ 0 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

and $x = \begin{bmatrix} e_y & e_a & \gamma & s \end{bmatrix}^T, u = \begin{bmatrix} \delta_f & \Delta M \end{bmatrix}^T, y = \begin{bmatrix} e_y & e_a \end{bmatrix}^T$ are the state variables vector, input signals vector, system output and uncertainties, respectively.

The control objective is to design a Model Predictive Control (MPC) law to track the desired trajectory (Fig. 1). It is worth emphasizing that the objective is not only required for trajectory tracking but also minimizing a given performance index with constraint set.

Remark 1. It is different from the traditional trajectory tracking control purpose in [1], which only consider the tracking error convergence without optimal control problem. Moreover, it should be noted that the satisfaction of tracking performance is difficult to obtain in MPC law due to the changing of control signal after each sampling time, especially in optimization with constraint.

III. MODEL PREDICTIVE CONTROL FOR A DISTURBED 4WID SYSTEM

In this section, a novel MPC law is proposed for a disturbed 4WID system under the Discrete Time System (DTS) consideration (see Fig. 3). At each time instant $l = k$ with time period T, the proposed MPC given in Eq. (8) is implemented during the time interval $\left[l,\ l+1 \right]$. The predictive model is given by employing the observer design. Additionally, the stability is guaranteed by considering the appropriate optimization problem with constraint set.

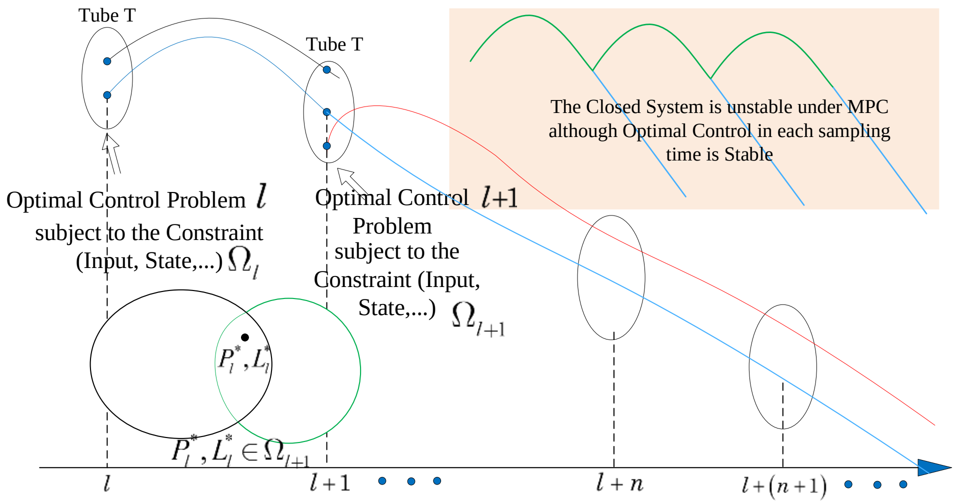

Fig. 3. Stability consideration in MPC.

Consider a time-invariant DTS as follows:

$$\begin{cases} x_{k+1} = Ax_k + Bu_k + \eta_{1,k} \\ y_k = Cx_k + \eta_{2,k} \end{cases} \tag{6}$$

where x_k, u_k, y_k are the state variable, input signal, output variable at time k, respectively. Additionally, $\eta_{1,k}$, $\eta_{2,k}$ are the disturbances of the first and second equations in Eq. (6).

Assumption 1: A is a Hurvitz matrix and there exists a matrix K such that $A+BKC$ is a Hurvitz matrix.

The disturbances $\eta_{1,k}, \eta_{2,k}$ are eliminated in Eq. (6) to obtain the nominal system as:

$$\begin{cases} z_{k+1} = Az_k + Bv_k \\ p_k = Cz_k \end{cases} \tag{7}$$

It follows that a tube-based RMPC law can be investigated as follows:

$$u_k = v_k + K\left(y_k - p_k\right) \tag{8}$$

where v_k is the MPC law to be designed later.

Define the states error between two systems Eqs. (6) and (7) $e_k = x_k - z_k$, we have:

$$\begin{aligned} e_{k+1} &= Ae_k + B\left(u_k - v_k\right) + \eta_{1,k} \\ &= \left(A + BKC\right)e_k + K\eta_{2,k} + \eta_{1,k} \end{aligned} \tag{9}$$

Using **assumption 1**, there exist two matrices $P = P^T > 0, Q > 0$ such that: $A_K^T P A_K - P = -Q$. Consider the Lyapunov function candidate: $V_k\left(e_k\right) = e_k^T P e_k$

According to Eq. (9), it implies that:

$$V_{k+1} - V_k = e_{k+1}^T P e_{k+1} - e_k^T P e_k$$

$$= \left(A_K e_k + \eta_k\right)^T P\left(A_K e_k + \eta_k\right) - e_k^T P e_k$$

$$\Rightarrow V_{k+1} - V_k = -e_k^T Q e_k + 2e_k^T A_K P \eta_k + \eta_k^T P \eta_k$$

where $\eta_k = K\eta_{2,k} + \eta_{1,k}$ and it is bounded by $\left\|\eta_k\right\| \le \eta_{\max}$. Therefore, it obtains that:

$$V_{k+1} - V_k \le -\lambda_{\min}\left(Q\right)\left[\begin{array}{l} \left(\left\|e_k\right\| - \dfrac{\left\|A_K P\right\|}{\lambda_{\min}\left(Q\right)}\eta^{\max}\right)^2 \\ -\left(\dfrac{\left\|A_K P\right\|^2 + \lambda_{\max}\left(P\right)\lambda_{\min}\left(Q\right)}{\lambda_{\min}\left(Q\right)^2}\right)\left(\eta^{\max}\right)^2 \end{array}\right] \tag{10}$$

and the attraction region can be obtained as:

$$\Omega = \left\{ e : \left\|e\right\| \le \frac{\left\|A_K P\right\| + \sqrt{\left\|A_K P\right\|^2 + \lambda_{\max}\left(P\right)\lambda_{\min}\left(Q\right)}}{\lambda_{\min}\left(Q\right)}\eta^{\max} \right\} \tag{11}$$

For each integer $i \ge 0$, denote the state at the time instant $k+i$ to be estimated at sampling time k by $\hat{\bullet}_{k+i|k}$.

In order to obtain the predictive model in considering MPC, we can employ the following state observer to estimate the state variable:

$$\begin{cases} \hat{z}_{k|k} = A\hat{z}_{k-1|k-1} + Bv_{k-1} + L\left(p_k - \hat{p}_{k|k-1}\right); \forall k \ge 1 \\ \hat{z}_{k+i|k} = A\hat{z}_{k+i-1|k} + Bv_{k+i-1}; \forall i \ge 1; k \ge 0 \\ \hat{p}_{k+i|k} = C\hat{z}_{k+i-1|k}; \forall i \ge 1; k \ge 0 \end{cases} \tag{12}$$

where $L = -BK$, $\hat{z}_{i|-1} = 0; \forall i \ge 0$

Based on Eq. (12), we obtain the dependence between the estimated value $\hat{z}_{k+i|k}$ and the control signal $v_{k+i|k}$. Next step, to obtain the unification between MPC and tracking problems, the cost function is modified as:

$$\begin{aligned} J_k &= \sum_{i=0}^{\infty}\left(\hat{z}_{k+i|k}^T R \hat{z}_{k+i|k}\right) + \sum_{i=0}^{m}\left(v_{k+i|k}^T Q_1 v_{k+i|k}\right) \\ &+ \sum_{i=1}^{m}\left(\Delta v_{k+i|k}^T Q_2 \Delta v_{k+i|k}\right) + \varepsilon_{k|k}^T S \varepsilon_{k|k} \end{aligned} \tag{13}$$

where $\Delta v_{k+i|k} = v_{k+i|k} - v_{k+i-1|k}; \forall i \ge 1$

$$\varepsilon_{k+i|k} = \hat{z}_{k+i|k} - \hat{z}_{k+i|k-1}; \forall i \ge 0; k \ge 0$$

The MPC law is given after solving the optimization problem with the corresponding constraint to guarantee the unification between MPC and tracking problem:

Problem 1: $\min_{\varepsilon_{k|k};v_{k|k};v_{k+1|k};...;v_{k+m-1|k}} J_k$

$$\text{subject to } \begin{cases} X_1 v_{k+i|k} + Y_1 \leq 0; \forall 0 \leq i \leq m \\ X_2 \Delta v_{k+i|k} + Y_2 \leq 0; \forall 1 \leq i \leq m \\ v_{k+i|k} = 0; \forall i \geq m+1 \\ G\hat{z}_{k+i|k} \leq g + \varepsilon_{k|k}; \forall i \geq 0 \\ \varepsilon_{k|k} \geq 0 \end{cases}$$

According to Eqs. (7) and (12), it implies the observer error $\mu_k = z_k - \hat{z}_{k|k}$ can be given as:

$$\mu_{k+1} = z_{k+1} - \hat{z}_{k+1|k+1} = (A - LC)\mu_k \tag{14}$$

From Eq. (12), it follows that $\varepsilon_{k|k} = \hat{z}_{k|k} - \hat{z}_{k|k-1}$ satisfies the equality as:

$$\varepsilon_{k+i|k} = A\left(\hat{z}_{k+i-1|k} - \hat{z}_{k+i-1|k-1}\right) = A\varepsilon_{k+i-1|k} \tag{15}$$

It should be emphasized that the traditional MPC is difficult to satisfy the tracking problem due to the changing optimization problem solution after each time period (see Fig. 3). Therefore, the advantage of the proposed MPC with an appropriate constraint can be known as the unification of optimization and tracking problem. Moreover, the tracking problem is also quantitative described in attraction region Eq. (11). To prove this determination, some following Lemmas are employed:

Lemma 1: For all squared matrix X , there exist two number ψ and $\kappa > 0$ such that: $\left\|X^n\right\| \leq \kappa\left(n^{\ell-1}\psi^n\right),\ \forall\ n \in \mathbb{N}$.

Lemma 2: There exists a positive number ξ such that:

$$\sum_{k=0}^{\infty}\left(\sqrt{\sum_{i=0}^{\infty}\varepsilon_{k+i|k}^T R \varepsilon_{k+i|k}} + \sqrt{\theta_k^T S \theta_k}\right) \leq \xi$$

Lemma 3: If the **optimization problem 1** is solved with $k = 0$ then there exists a solution $\varepsilon_{k|k}^*; v_{k|k}^*; v_{k+1|k}^*; ...; v_{k+m-1|k}^*$ of the **problem 1** for each $k \in \mathbb{N}$ and the optimal function is satisfied as $J_k^* \leq \left(\sqrt{J_0^*} + \xi\right)^2$ with ξ being obtained in **lemma 2**.

Based on three above Lemmas, the following Theorem fulfills the unification between optimization and stability problems

Theorem 1: The MPC law is the output feedback control v_k , which is the first value of the sequence $\left\{v_{k|k}; v_{k+1|k}; ...; v_{k+m-1|k}\right\}$ being the solution of **optimization problem 1,** can be guaranteed for the stability of the nominal system Eq. (7).

Proof:

In the view of **lemma 3**, it implies that:

$$\begin{aligned} J_{k+1}^* &\leq J_k^* - \left|\hat{z}_{k|k}\right|_R^2 + \left(\sqrt{\sum_{i=1}^{\infty}\left(\left|\varepsilon_{k+i|k+1}\right|_R^2\right)} + \left|\theta_k\right|_S^2\right)^2 \\ &+2\left(\sqrt{J_0^*} + \xi\right)\left(\sqrt{\sum_{i=1}^{\infty}\left(\left|\varepsilon_{k+i|k+1}\right|_R^2\right)} + \left|\theta_k\right|_S^2\right) \end{aligned} \tag{16}$$

$$\begin{aligned} \Rightarrow J_{k+1}^* + \sum_{i=0}^{k}\left|\hat{z}_{i|i}\right|_R^2 &\leq \sum_{m=0}^{k}\left(\sqrt{\sum_{i=1}^{\infty}\left(\left|\varepsilon_{m+i|m+1}\right|_R^2\right)} + \left|\theta_m\right|_S^2\right)^2 \\ &+2\left(\sqrt{J_0^*} + \xi\right)\sum_{m=0}^{k}\left(\sqrt{\sum_{i=1}^{\infty}\left(\left|\varepsilon_{m+i|m+1}\right|_R^2\right)} + \left|\theta_m\right|_S^2\right) \end{aligned} \tag{17}$$

$$\Rightarrow s_k = \sum_{i=0}^{k}\left|\hat{z}_{i|i}\right|_R^2 \leq \xi^2 + 2\left(\sqrt{J_0^*} + \xi\right)\xi; \forall k \geq 0$$

Therefore, as $k \to \infty$, we have $\left|\hat{z}_{k|k}\right|_R^2 = s_k - s_{k-1} \to 0$, i.e., $\left\|\hat{z}_{k|k}\right\| \to 0$.

IV. Simulation Results

Consider a 4WID with the parameter to be chosen as follows:

$$m = 1416\ (kg), d = 1.8\ (m), l_f = 1\ (m), l_r = 1.5\ (m)$$
$$J = 0.9\ (kg \cdot m^2), R = 0.311\ (m), I_z = 1500\ (kg \cdot m^2),$$
$$C_f = C_r = 26000\ (N/rad).$$

To develop the MPC design for the unification between optimization and tracking problem, the following weight matrices in cost function Eq. (13) are chosen as: $R = \begin{bmatrix}1 & 0\\ 0 & 3\end{bmatrix}$; $Q_1 = 1$; $Q_2 = 2$; $S = 5$ and the sampling time is $T_s = 0.1$. It should be noted that the selection of these above matrices requires the satisfaction of symmetric and positive definition property. The feedback gain matrix K is given by Acker function in Matlab software. Moreover, the solution of the proposed optimization 1 is implemented by Yalmip tool, which is able to obtain the solution approximation.

The desired trajectory is selected in two scenarios, including the straight line and the curve line to validate the effectiveness of the proposed MPC method. It can be seen that, Figs. 4–6 show the tracking results of the proposed MPC method in the 4WID although there exists a change of the direction. In this scenario, the vehicle follows the straight line and curve line references under the control signals in Figs. 5 and 7. Additionally, it is worth noticing that the control signals in Figs. 5 and 7 satisfy the constraint set as described in the proposed optimization 1. In addition to the practical systems, the control signals in Figs. 5 and 7 can be developed by micro controller. Therefore, Figs. 4–7 show the unification of tracking performance and MPC problem, which is difficult to guarantee due to the changing of control signal after time interval.

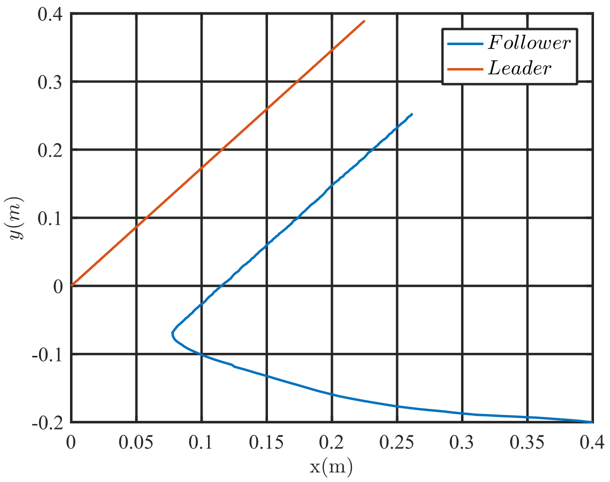

Fig. 4. The tracking behavior of 4 WID with the reference being the straight line.

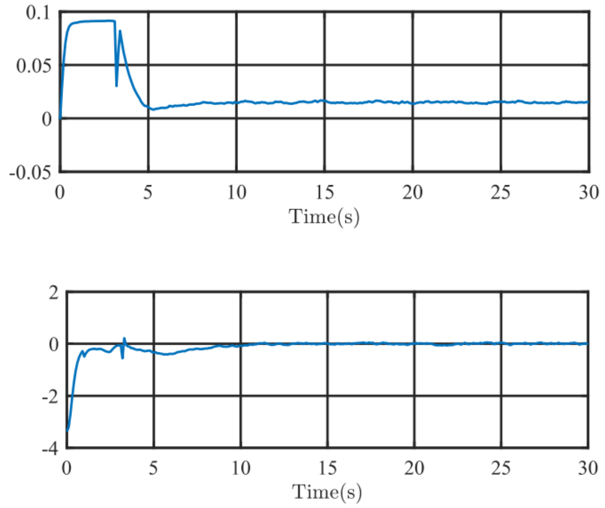

Fig. 5. The control signals $u = [\delta_f \quad \Delta M]^T$ with the reference being the straight line.

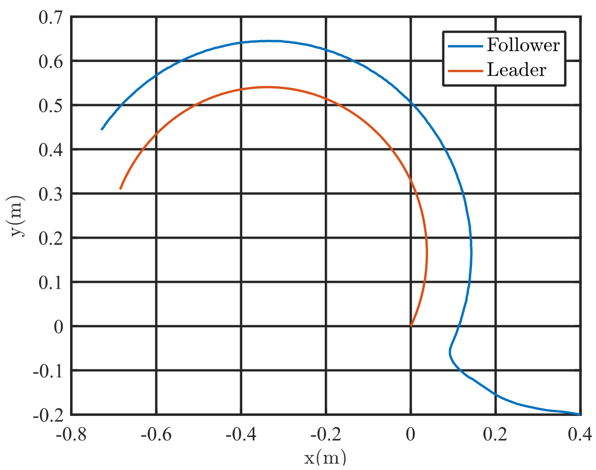

Fig. 6. The tracking behavior of 4 WID with the reference being the Curve line.

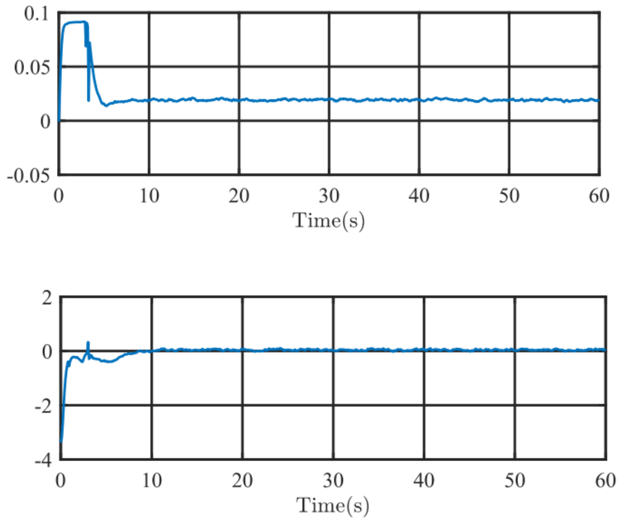

Fig. 7. The control signals $u = [\delta_f \quad \Delta M]^T$ with the reference being the Curve line.

V. Conclusion

In the study, a novel MPC strategy has been established for a nonlinear FWID system to ensure not only the optimization with constraint but also trajectory tracking performance. To handle the challenge of nonlinear model, the linearization method is developed to implement LMI based optimization problem at each time instant. An output feedback MPC design methodology is proposed for the construction of a state observer and an optimization problem. The satisfaction of tracking performance and optimization is guaranteed by the inequality estimations without the consideration of terminal control as well as terminal region as the previous researches. Furthermore, the feasibility is proved by both theoretical analysis and simulation results, which are different from existing MPC strategies to be considered by searching the terminal region and terminal control. Finally, our future research direction is to develop MPC for Multiagent Systems (MASs) with complete dynamic model of each agent.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Ngoc Son Tran wrote the paper, conducted the simulation, Khac Lai Lai wrote the draft paper and Phuong Nam Dao wrote the draft paper, reviewed the paper. All authors had approved the final version.

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