



Research Paper

A COMPUTER BASED TECHNIQUE FOR STRUCTURAL COMPARISON OF IN PARALLEL ROBOTIC MANIPULATORS APPLICABLE FOR HIGHER PAIRS

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Kinematic linkages play a very vital role in the motion/power transmission and as well in the generation of various workspaces/functions which in turn affect the type and nature of work done by a given linkage. The usage of kinematic linkages is thus very much necessary in the robot manipulators so as to obtain the necessary motion of the end-effectors to perform various tasks assigned to the robot. The robot manipulator linkages are till date being generated and selected based on the trial and error and there is as such no measure to decide or fix which one of the available linkages is the most appropriate one for a specified task of the end-effector. Detection of similarity between two or more linkages (Isomorphism) is very much important in this regard so that before using the optimizing techniques itself we reduce the number of linkages being considered for a fixed task to a minimal level, and Distinct number technique is very much is a step towards this way. Isomorphic linkages of same configuration and their analyses is based on the equations governing the size of workspace and the condition of workspace, with the help of a 'c' programme an attempt is made for this purpose. The output of these programs or the programs as a whole can be utilized by robot manufacturing company for choosing the optimum kinematic linkage for the end-effectors of the manipulator for the robot being manufactured by them to suite a specific purpose which in turn shall satisfy the requirements and becomes a less time consuming process as well as economic for the manufacturer and can even improve the efficiency of the end-effectors.

Keywords: Isomorphism, Distinct number, Adjacency, Graph

INTRODUCTION

The synthesis of a kinematic chain is the most important aspect while choosing it as a

manipulator for a robot or to choose such kinematic chain as a mechanism for a machine component. Almost all the study reported on

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structural aspects of linkages and kinematic chains pertain to generation of distinct kinematic chains. All the relevant studies reported so far would not have much significance, if quantitative methods are not developed to compare all the distinct kinematic chains with the same number of elements and degrees-of-freedom for different aspects. It is always desirable to know the anticipated behavior of the kinematic chains without actually design and test on them. At present, the designer has to depend upon intuition to select the best possible kinematic chain and this may not always lead to optimum results. Hence quantitative methods, simple and less time consuming are needed to compare the kinematic chains at the conceptual stage of design it self.

In this work Mathematical technique called distinct number technique is discussed. In the process of graph or chain generation, the isomorphic graphs are some times encountered. These are kinematic chains mathematically distinct but are kinematically identical. So there is a need to select one among them. It has been a cumbersome task to identify manually such isomorphic chains among numerous kinematic chain inversions. Distinct number technique is useful to make the above process easier. Case studies have been done by taking few kinematic chains of interest and their results are discussed.

A programme in 'C' have been generated for the above rating techniques and executed so as to reduce the human errors in calculating the above ratings and to reduce the work load on the user, especially when the no.of links are higher in number.

DESCRIPTION OF TERMINOLOGY USED IN THE PAPER

Isomorphism

If two kinematic chains are functionally similar then they are said to be isomorphic in nature.

For a specified number of links in a kinematic chain, there may be a number of inversions if the type of pairs in them are altered, i.e. for a particular number of links one can get a number of inversions if every pair is interchanged with a higher one and a lower one and vice versa.

But from them choosing one chain for a given purpose is not so easy task and there may be some isomorphic chains among them. So there is a need to classify them and divide the isomorphic chains. So that, one among the isomorphic chains can be selected, that would be the best one for the purpose. If this process can be done manually, it is laborious and the resources needed are very high.

So, to identify the isomorphic chains from all the possible inversions of a given number of linkages in a kinematic chain, a theoretical method was developed which reduces the initial cost of the design process to a great extent.

Kinematic Synthesis

The design or creation of a mechanism to yield the desired set of motion characteristics is called kinematic synthesis. Synthesis may be classified in broad as.

- Type Synthesis: This is the beginning phase of the mechanism. It refers to the kind of mechanism selected. It might be a linkage,

a geared system, belts, pulleys or a cam system.

- **Number Synthesis:** It deals with the number of links, joints or pairs required to obtain certain mobility, the study of mobility of a mechanism in terms of degree-of-freedom (d.o.f.).
- **Dimensional Synthesis:** It deals with the determination of actual dimensions (lengths, angles, etc.) of the mechanism to satisfy the specified motion characteristics.

Manipulator

A manipulator is a mechanical motion device used to access any point within the workspace. Any industrial robot cannot exist with out a

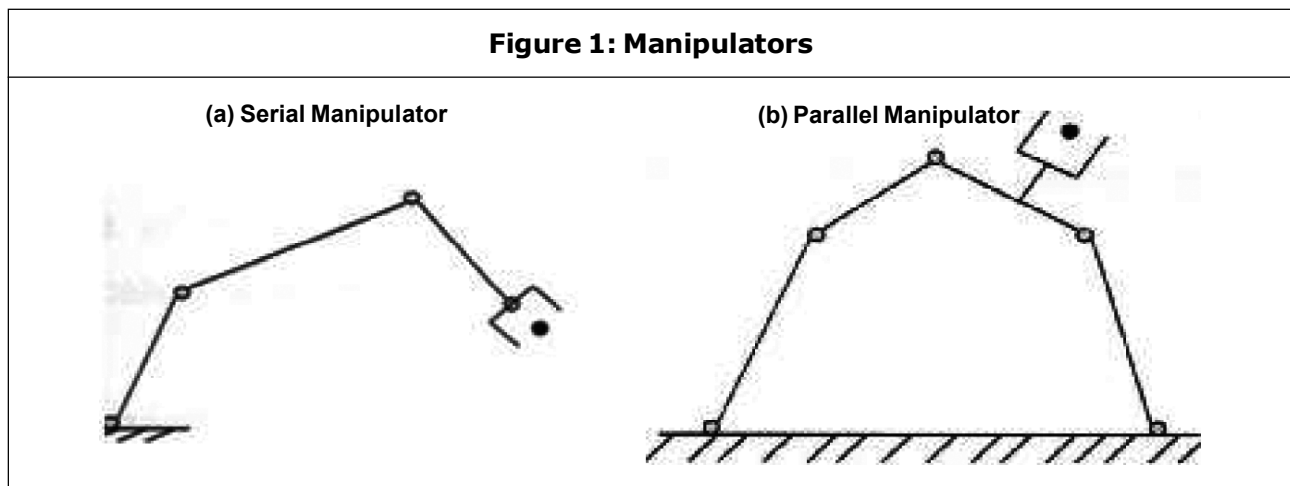
manipulator. A manipulator can generally be described as a kinematic chain.

Manipulators in general are of two types.

1. Serial manipulators
2. Parallel manipulators

Serial manipulators consist of a single chain of links and joints connecting the tool to ground.

Parallel manipulators are closed-loop mechanisms in which the end-effector, is connected to the base by at least two independent kinematic chains. These consist of multiple load-bearing paths between the tool and ground (Figure 1).



Workspace of a Manipulator

The workspace of a manipulator is defined as the volume of space in which the manipulator is able to locate its end effectors. Size of workspace depends on the configuration of the manipulator, size of the links and wrist joints.

Workspace can generally be classified as:

- Reachable workspace
- Dexterous workspace

Reachable Workspace

This is the region that can be reached by the origin of the end-effectors frame with at least one orientation.

Dexterous Workspace

The region or space where the end effectors can reach every point in more than one orientation is called dexterous workspace.

In general a dexterous workspace is a subset of a reachable workspace.

Requirements of a Robot Manipulator

- A manipulator has the ability to access any point within its workspace.
- A manipulator must be able to generate accurate path/function.
- A manipulator must generate well conditioned workspace.

Function Generation

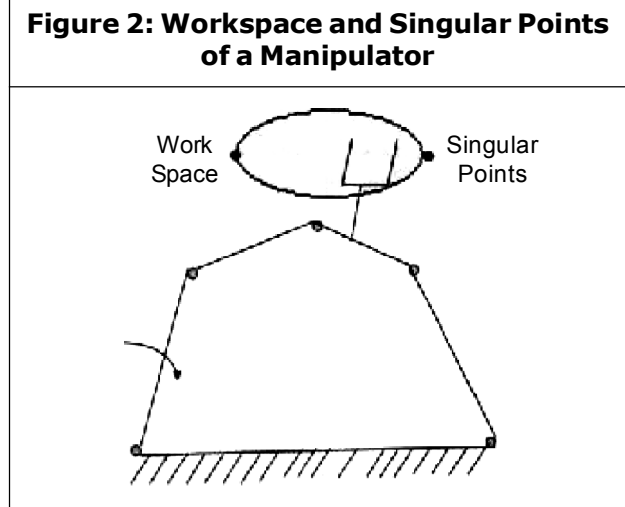
The requirement in design is that causing an output member to rotate, oscillate, or reciprocate to a specified function of time or function of input motion. This is called generation of function.

Path Generation

The coupler of the mechanism may be made to move along a part or full portion of a circular, linear or an arbitrary curve, which is meant that the output is generating a particular path.

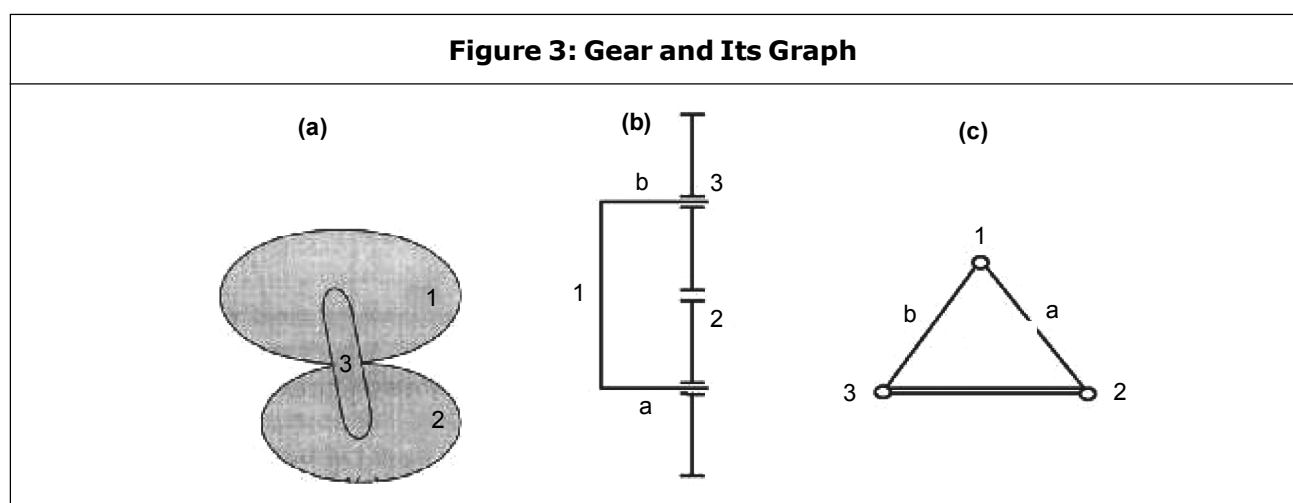
Singular Points

These are the positions in the kinematic linkage where it loses one or more degrees of freedom. These are generally generated when any two binary links in a kinematic chain comes in collinear (Figure 2).



GRAPH REPRESENTATION

It is easy to deal a kinematic structure especially a gear train when it is represented by a graph. A graph consists of vertices and the edges that join the vertices. A link or an element of a gear train is represented by a vertex: a small circle. A gear pair or joint in gear trains is represented by an edge. Gear trains consist of two types of pairs (i) turning pairs and (ii) gear pairs. A turning pair is represented by a single line edge while a gear pair is represented by a double line edge. For example, a simple gear train of Figure 3a whose schematic representation (known as



Levoi notation) is shown in Figure 3b and by the graph in Figure 3c. In Figure 3b, element 1 is the carrier and elements 2 and 3 are the gear wheels.

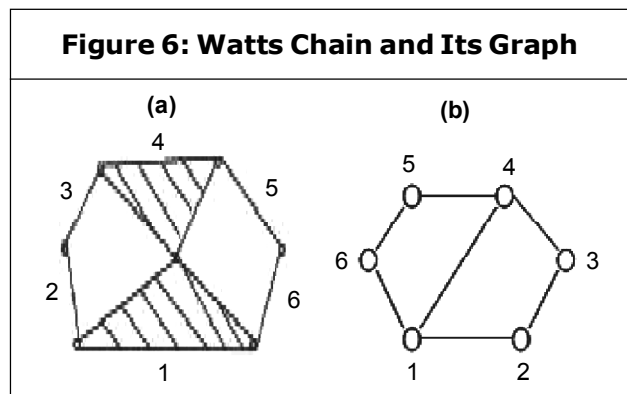
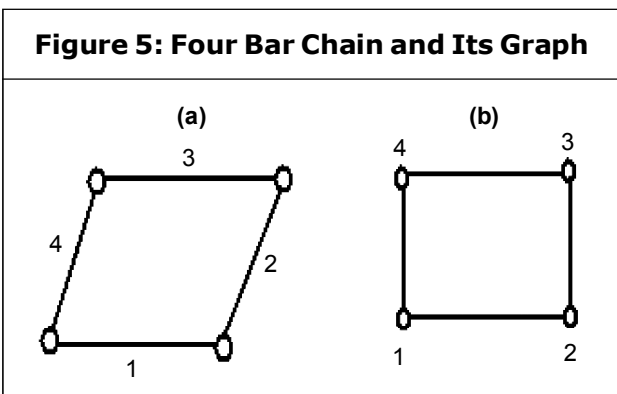
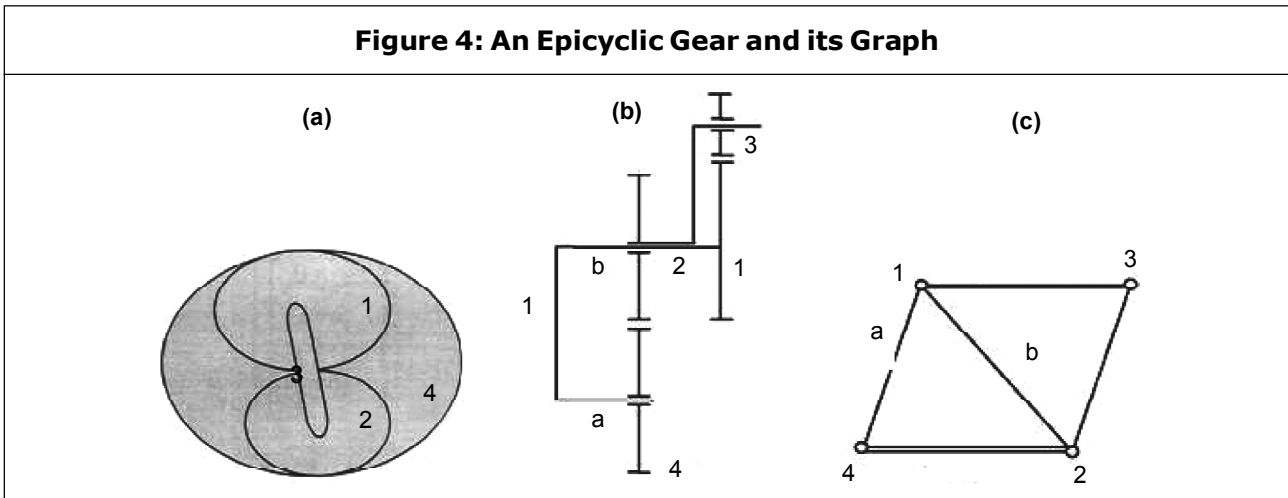
The turning pair edges 1-2 and 1-3 of Figure 3c are labeled as a and b corresponding to the levels of the gear wheels 2 and 3 on the carrier.

Let us consider another example to understand graph theory. The gear train shown Figure 3a has been added with an outer envelope of gear '4' which is shown in Figure 4a. Its schematic/Levoi notation is shown in Figure 4b. The graph of this epicyclic gear train shall be obtained by adding a gear pair between vertex 4 as well as another gear pair between vertex 2 and

4. Therefore there shall be 2 gear pairs at either of the vertices 1 and 2.

Similarly a kinematic chain can also be represented by its graph as shown in Figures 5a and 5b; where Figure 5a represents a four bar chain which has four links 1, 2, 3, 4. To convert the linkage Figure 5a into a graph of Figure 5b all the links have to be converted in to vertices and the type of pair between each links of Figure 5a will decide the type of the connectivity between these vertices, i.e., if two links are joined by a turning pair then their corresponding vertices shall be joined using a single line.

In the same manner the watts chain of Figure 6a can be converted in to its graph as shown in Figure 6b.



DISTINCT NUMBER TECHNIQUE

Different techniques available for testing structural isomorphism in kinematic chains are:

- Distance matrix method;
- Linkage characteristic polynomial;
- Edge permutation group method;
- Standard code technique; and
- Acyclic graph method.

In order to avoid confusion with larger chains, computer aided methods, to test isomorphism are needed. The method used here is distinct number technique which can be applied to kinematic linkages, gear trains, cam mechanisms, hydraulic piston cylinder mechanisms, and spring mechanisms, etc.

Distinct Number Technique

By using the graph of any kinematic chain drawn using the above mentioned procedure, this technique can be applied.

This technique involves 3 phases:

1. Generation of Adjacency matrix.
2. Framing of distinct matrix from Adjacency matrix.
3. Calculation of distinct number and distinct string from distinct matrix.

Phase 1

To frame Adjacency matrix we need to understand the no of vertices in the graph of the kinematic chain being considered; for instance let us consider the graph of four bar linkage showed in Figure 5a, there are four vertices hence the Adjacency matrix shall be

a 4×4 matrix. Thus for any kinematic chain the Adjacency matrix and hence the distinct matrix shall be $n \times n$ matrix. Where n denotes the number of vertices.

Framing of Adjacency matrix can be clearly understood using an example .let us consider the four bar linkage of Figures 5a and 5b whose Adjacency matrix shall be 4×4 matrix and every element in a given row of the matrix shall denote the correlation between that particular row number vertex (for example: first row signifies first vertex, second row signifies second vertex...). Thus the first row first element shall signify the type of connectivity between first vertex to itself; since any vertex cannot connect with itself this particular element shall be assigned a value zero (0). Thus all the diagonal elements which signify the connection between n^{th} vertex to itself shall be zero (0). The first row second element shall be framed by correlating the type of connectivity between vertex 1 and 2. Since in Figure 5b vertex 1 and 2 are connected using turning pair which is a lower pair according to fundamentals of kinematics a we assigns a value of 1. Similarly if there is any higher pair (gear pairs) connectivity between any vertices then we assign a value of 2.

$$\begin{matrix} & 1 & 2 & 3 & 4 \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \left(\begin{array}{cccc} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{array} \right) \end{matrix}$$

The above framed matrix is the Adjacency matrix for a four bar linkage. Similarly the Adjacency matrix of watts chain represented in Figure 6a can be framed as below.

$$\begin{matrix} & 1 & 2 & 3 & 4 & 5 & 6 \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \left(\begin{matrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{matrix} \right) \end{matrix}$$

The Adjacency matrix signifies the Adjacency between any two vertices. For instance the element of second row third position in a watts chain shall tell the user that what will be the type of connectivity between vertex 2 and 3. At the same time it shall also mention the user whether there will be a connectivity or not between any two given vertices.

Phase 2

The Adjacency matrix fails to signify any specific aspects which can compare two similar kinematic chains. This draw back can be over come by using the distinct matrix. Which is framed by comparing n^{th} row, $(n + 1)$, $(n + 2)$, ... rows. This matrix shall also be having same number of rows and columns as that of Adjacency matrix hence. By comparing the first row with itself shall specify the value of first row first element in a distinct matrix. Similarly first row second element shall be framed out of comparison of first and second rows and like wise the matrix can be framed. Here we follow the rule that when ever we come across two similar elements at i^{th} position of the two rows been compared we take the value as zero; if we are encountered with two dissimilar elements at any i^{th} position of the two rows being compared we take the sum of

these two iih elements. Finally we can frame any i^{th} element of a distinct matrix by taking the algebraic sum of all the elemental comparison as discussed above.

$$\left(\begin{matrix} 0 & 5 & 1 & 6 & 1 & 5 \\ 5 & 0 & 4 & 1 & 4 & 2 \\ 1 & 4 & 0 & 5 & 2 & 4 \\ 6 & 1 & 5 & 0 & 5 & 1 \\ 1 & 4 & 2 & 5 & 0 & 4 \\ 5 & 2 & 4 & 1 & 4 & 0 \end{matrix} \right)$$

Phase 3

All the row elements in distinct matrix shall be added together to give the distinct value of that particular row. For example $16 + 16 + 16 + 16 + 18 + 18 = 100$ is the distinct value of second row of distinct matrix for watts chain. Like wise all this distinct values shall be algebraically added together to obtain the distinct number. To designate the distinct number and distinct values (which are combined to called as distinct string). We give the relation as shown below.

$$DI [A, B, C, D, E, F]$$

In the above notation DI stands for distinct number which is the total of all the distinct values and the terms mentioned in square brackets are the distinct values arranged either in ascending or descending order. For example distinct number for watts chain is:

$$100 [16, 16, 16, 16, 18, 18]$$

Isomorphism from Distinct Number

Two kinematic chains can be compared for their similar arrangement using their distinct numbers. For example the distinct no of Watt's chain is $100 [16, 16, 16, 16, 18, 18]$ and the

Stephenson’s chain is 100 [14, 14, 16, 16, 20, 20]. This tells us that on comparison these two chains are not identical because even though their distinct numbers are same the distinct strings are dissimilar. Thus we can mention that any two chains are said to be isomorphic only if their distinct numbers along with all their corresponding distinct string elements are equal.

Algorithm for the programme developed:

Step 1: Input is taken for number of linkages to be compared.

Step 2: Input is taken for number of links present in each linkage.

Step 3: Input is taken for relative motion between a link with every other link.

Step 4: Adjacency matrix for each linkage is developed using logic conditions (as stated in Phase 1).

Step 5: Distinct matrix for each linkage is developed using logic conditions (as stated in Phase 2).

Step 6: For every row of elements present in distinct matrix, sum of all the elements in each row is determined and arranged in ascending order (as stated in Phase 3).

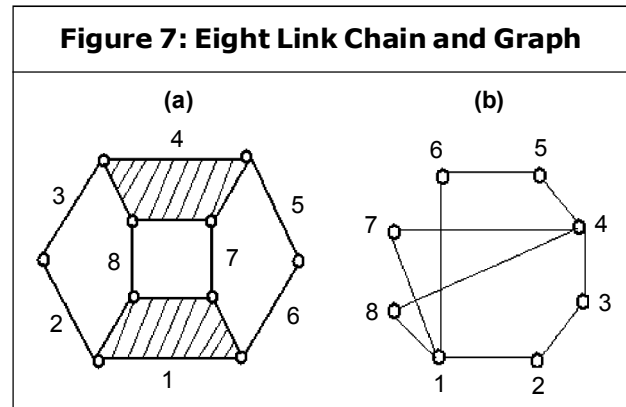
Step 7: Distinct number for each linkage is determined by adding all the elements present in distinct matrix (as stated in Phase 3).

Step 8: Distinct number and distinct string of each linkage is compared with every other linkage and if they are same they are said to be “isomorphic”, if not said to be “non-isomorphic”.

Results of 8 link single degree of freedom chains using the programme:

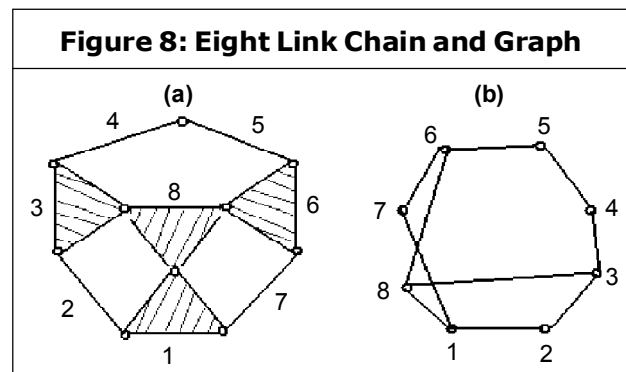
Let us consider Linkage 1 is as shown in Figure 7a.

The graph theory for the linkage is as shown in the Figure 7b.



Let us consider Linkage 2 is as shown in Figure 8a:

The graph theory for the linkage is as shown in the Figure 8b.



C Programme (See Programme Input and Output)

```
#include<stdio.h>
#include<conio.h>
main()
{
int a[64][64],b[64][64],a1[64][64],b1[64][64],a2[64],b2[64];
int i,j,k,n;
```

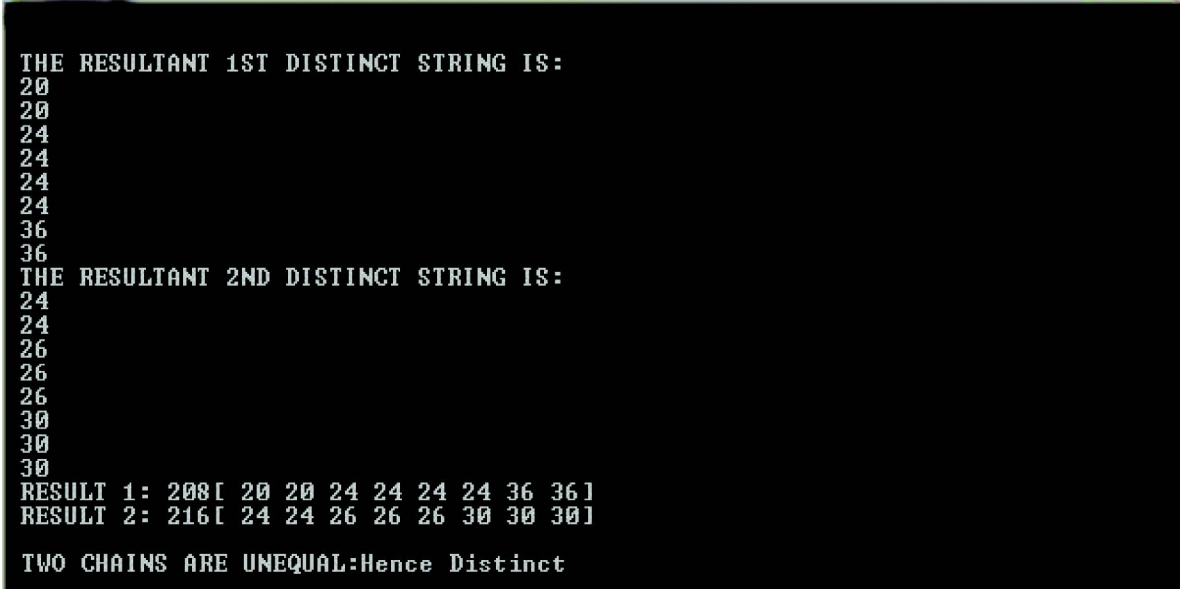

Programme Input and Output

```
C:\TCVTC.EXE
ENTER THE ORDER OF THE MATRICES=8
ENTER THE FIRST MATRIX ELEMENTS:
M I N N M I | I
1 0 1 0 0 0 0 0
0 1 0 1 0 0 0 0
0 0 1 0 1 0 1 1
0 0 0 1 0 1 0 0
1 0 0 0 1 0 0 0
1 0 0 1 0 0 0 0
1 0 0 1 0 0 0 0
```

```
C:\TCVTC.EXE
ENTER THE SECOND MATRIX ELEMENTS:
0 1 0 0 0 0 1 1
1 0 1 0 0 0 0 0
0 1 0 1 0 0 0 1
0 0 1 0 1 0 0 0
0 0 0 1 0 1 0 0
0 0 0 0 1 0 1 1
1 0 0 0 0 1 0 0
1 0 1 0 0 1 0 0_
```

```
C:\TCVTC.EXE
AFTER PROCESS :THE RESULTANT 1ST MATRIX:
06444666
60444222
44062422
44606466
44260422
62444022
62262200
62262200

AFTER PROCESS:THE SECOND RESULTANT 2ND MATRIX:
05255256
50524521
25053456
52504343
54340523
25435056
52542501
61633610
```



```

C:\ATC\TC.EXE
THE RESULTANT 1ST DISTINCT STRING IS:
20
20
24
24
24
24
36
36
THE RESULTANT 2ND DISTINCT STRING IS:
24
24
26
26
26
30
30
30
RESULT 1: 208[ 20 20 24 24 24 24 36 36]
RESULT 2: 216[ 24 24 26 26 26 30 30 30]
TWO CHAINS ARE UNEQUAL:Hence Distinct

```

```

int sum1=0,sum2=0,count=0;
int temp,e1=0,e2=0;
clrscr();
printf("\n ENTER THE ORDER OF THE
MATRICES:");
scanf("%d",&n);
printf("\n ENTER THE FIRST MATRIX
ELEMENTS:");
    for(i=0;i<n;i++)
{
    for(j=0;j<n;j++)
        {
            scanf("%d",&a[i][j]);
        }
        printf("\n");
}
printf("\n ENTER THE SECOND MATRIX
ELEMENTS:");
    for(i=0;i<n;i++)
    {
        for(j=0;j<n;j++)
        {
            for(k=0;k<n;k++)

```

```

        if(a[i][k]!=a[j][k])
            e1++;
        if(b[i][k]!=b[j][k])
            e2++;
    }
    a1[i][j]=e1;
    e1=0;
    b1[i][j]=e2;
    e2=0;
}
}
printf("\n AFTER PROCESS :THE
RESULTANT 1ST MATRIX:\n");
for(i=0;i<n;i++)
{
    for(j=0;j<n;j++)
    {
        printf("%d",a1[i][j]);
    }
    printf("\n");
}
printf("\n AFTER PROCESS:THE
SECOND RESULTANT 2ND MATRIX:");
for(i=0;i<n;i++)
{
    for(j=0;j<n;j++)
    {
        printf("%d",b1[i][j]);
    }
    printf("\n");
}
}
for(i=0;i<n;i++)
{
    a2[i]=0;
    b2[i]=0;
    for(j=0;j<n;j++)
    {
        a2[i]=a2[i]+a1[j][i];
        b2[i]=b2[i]+b1[j][i];
    }
}
getch();
for(i=0;i<n-1;i++)
{
    for(j=i+1;j<n;j++)
    {
        if(a2[i]>=a2[j])
        {
            temp=a2[i];
            a2[i]=a2[j];
            a2[j]=temp;
        }
        if(b2[i]>=b2[j])
        {
            temp=b2[i];
            b2[i]=b2[j];
            b2[j]=temp;
        }
    }
}
}

```

```

}
temp=0;
for(i=0;i<n;i++)
{
    sum1=sum1+a2[i];
    sum2=sum2+b2[i];
}
printf("\n THE RESULTANT 1ST
DISTINCT STRING IS:");
for(i=0;i<n;i++)
{
    printf("\n %d",a2[i]);
}
printf("\n THE RESULTANT 2ND
DISTINCT STRING IS:");
for(i=0;i<n;i++)
{
    printf("\n %d",b2[i]);
}
printf("\n RESULT 1:%4d[" ,sum1);
    for(i=0;i<n;i++)
    {
        printf("%3d",a2[i]);
    }
    printf("]");
printf("\n RESULT 2:%4d[" ,sum2);
    for(i=0;i<n;i++)
    {
        printf("%3d",b2[i]);
    }
}
printf("]");
for(i=0;i<n;i++)
{
    if(a2[i]==b2[i])
        count++;
}
if(count==n)
    printf("\n\n TWO CHAINS ARE
EQUAL:Hence Isomorphic");
else
    printf("\n\n TWO CHAINS ARE
UNEQUAL:Hence Distinct");
printf("\n press any key to continue..");
getch();
}

```

CONCLUSION

There are around 360 arrangements for a single degree of freedom 7 link kinematic chain. Like wise for any degree of freedom and number of links there will be numerous possible arrangements. But out of all these arrangements many may be similar to one another which cannot be visualized or identified while generation. The method proposed is very much useful for the designer who is so far being burdened with the tedious task of selecting a particular kinematic chain with a particular number of links. This method is further enhanced for its utility by using the computerized programming developed by the authors who are very user friendly and can generate, detect, rate any configuration of kinematic chains with a maximum limit up to 30 links with single degree of freedom. Thus

this work is very futuristic, user friendly and is useful for any designer of a gear train or a robot manipulator. 🌀

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