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Research Paper

OPTIMIZATION OF RELATIVE POSITIONING IN A TWO WHEELED DIFFERENTIAL DRIVE ROBOT

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Pose estimation for mobile robots depends basically on accurate odometry information. Odometry from the wheel's encoder is mostly used for simple and inexpensive implementationfor determining the relative position of a mobile robot. This paper deals with the determination of better relative localization of a two wheeled differential drive robot by means of odometry by considering the influence of parameters namely payload, speed, diameter of wheel and thickness of wheel. Experiments have been conducted based on central composite rotatable design matrix. A mathematical model has been developed for the robot using Response Surface Methodology (RSM) with the help of MINITAB software. An optimum relative positioning was obtained by using Genetic Algorithm (GA).

Keywords: Mobile robot, Odometry, Relative localization, Response surface methodology, Genetic algorithm

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INTRODUCTION

Navigation is a key ability of mobile robots and the task of navigation can be divided into localization and path planning. The objective of localization is to determine the position of a mobile robot in its environment, given a map of the environment and local sensorial data. Pose estimation of robot has been known as one of the most fundamental problems in mobile robotics (Byrne *et al.*, 1992; and Julian Lategahn *et al.*, 2010). Two basic pose estimation methods (absoluteand relative positioning) are widely employed in mobile robots (Hollingum, 1991; Chenavier and Crowley, 1992; and Evans, 1994). Odometry is the mostly used navigation method for relative positioning of mobile robot because it provides good accuracy, is inexpensive, and allows very high sampling rates (Borenstein, 1998).

Absolute positioning methods usually rely on navigation beacons, active or passive landmarks, map matching and satellite based navigation signals but none of these existing

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methods are well designed. Another approach to the pose determination of mobile robots is based on inertial navigation with gyros and accelerometers. The experimental results acquired by the researchers (Barshan and Durrant-Whyte, 1994; and Borenstein and Feng, 1996) indicate that this approach is not advantageous. Gyros can be more accurate and costly but they provide information only on the rate of rotation of a robot (Komoriya *et al.*, 1994). This sort of problem does not exist with electronic compasses that measure the orientation of the robot relative to magnetic field of earth. However, electronic compasses are not recommended for indoor applications due to the large distortions of the magnetic field near power lines or steel structures (Byrne *et al.*, 1992).

Many researchers have concentrated upon the accurate calibration of odometry (Krantz, 1996; and Kooktae Lee *et al.*, 2011), measuring errors in odometry and development of models for minimizing the odometry errors (Lauro Ojeda and Borenstein, 2004; and Korayem *et al.*, 2006) in wheeled robots. In addition, researchers have focused on the optimum path planning in mobile robots (Shen Zhi-cua *et al.*, 2006; and Gonzalez-Gomez *et al.*, 2011) and stability analyses in mobile robots (Eghtesad and Necsulescu, 2004; Chaoli Wang *et al.*, 2010; Jianxian Cai and Xiaogang Ruan, 2011; and Yao Cai *et al.*, 2012).

Robot's movement and abilities on particular terrain are affected by many factors like geometry and type of locomotion system (wheeled, tracked, hybrid, legged, jumping), properties of effectors (e.g., tyre type for wheeled robots), mass properties of a robot and constraints resulting from characteristics of drives (Maciej Trojnacki, 2012).

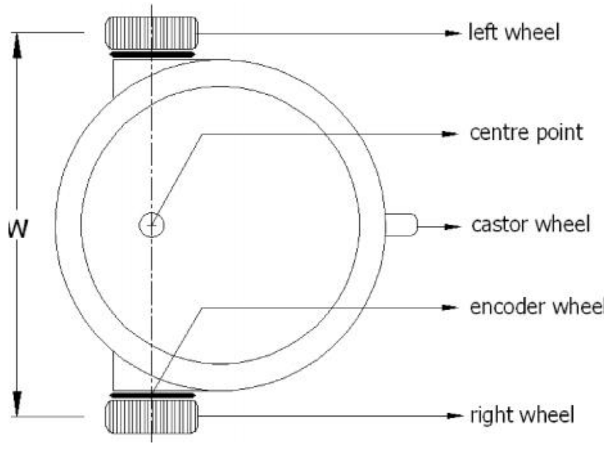
From the literature, it is observed that the major research have been focused on the development of odometry error models, stability analyses and path planning for mobile robots. Very few researchers have considered the effect of parameters like payload, velocity and geometry of wheels formobile robot in the determination of relative localization using odometry. In this paper,a mathematical model for odometry error of a two wheeled differential drive robot has been developed using RSM and an optimum condition was obtained through GA.

TWO WHEELED DIFFERENTIAL DRIVE ROBOT

A two wheeled differential drive robot "VENTRA" shown in Figure 1 was employed for the conduct of experiments in this study. The robot was driven in an indoor environment for a distance of 2 m in a straight line path. An evenly paved cement concrete floor was used as terrain which normally minimizes the chance of non-systematic errors such as wheel slippage,interaction with external bodies and travel over unexpected objects on the floor.

The self weight of the robot is 1.2 kg and the distance between two wheels (W) is 120 mm. The maximum speed is upto 200 mm/s. Two encoder wheels with encodersare used to calculate the linear displacement of each wheel. The new orientation of the robot can be estimated from difference in encoder counts, diameter of the wheels and distance between the wheels.

Figure 1: VENTRA Robot



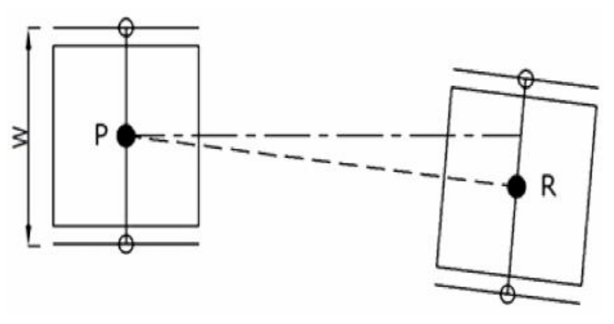
Calculation of Odometry Error

Odometry is a measuring method of wheel rotation as a function of time. If the two wheels of the robot are joined to a common axle, orientation of the centre of the axle relative to the previous orientation can be determined from odometry measurements on both wheels. In actual practice, optical encoders mounted on both wheels feed discretised wheel increment information to the controller, which in turn used to calculate the robot's state using geometric equations.

The wheel base (W) of the robot is the space between the contact points of two rear wheels. The center of the robot with respect to odometry is the midpoint between these two contacts. To calculate the variation in position and orientation of the robot with respect to starting point (P) across a given span of time, linear distance DR and DL of each wheel traveled (computed from number of ticks of the encoders and diameter of the wheels) and wheel base (W) are substituted in the following Equation (1). The new orientation (R) in radians shown in Figure 2 is calculated by

$$R = P + (D_R - D_L)/W \quad \text{...(1)}$$

Figure 2: Orientation of Robot



RESPONSE SURFACE METHODOLOGY

Response Surface Methodology (RSM) is one of the effective method of analyzing the result of a factorial experiment. Odometry error (Oe) can be treated as output response and expressed as a function of parameters namelypayload (L), speed (S), diameter of wheel (D) and thickness of wheel (T) as indicated in Equation (2).

$$\text{Odometry error } (Oe) = \Phi(L_{iu}, S_{iu}, D_{iu}, T_{iu}) + e_u \quad \text{...(2)}$$

where, Φ = response surface, e_u = residual, u = number of observations in the factorial experiment and iu represents level of the i^{th} factor in the u^{th} observation. When the mathematical form of Φ is unknown, it can be approximated by polynomials satisfactorily within the experimental region in terms of parameter variables. The ranges of all the parameters were fixed by conducting trial runs. This was performed by varying one of the parameters while keeping the rest of them as constant values. The upper limit of a given parameter was coded as (+2) and the lower limit was coded as (–2). The coded values for intermediate values were calculated using the Equation (3).

$$X_i = \frac{2(2X - (X_{max} + X_{min}))}{(X_{max} - X_{min})} \quad ...(3)$$

where

X_i : Required coded value of a variable X

X : Any value of the variable from X_{min} to X_{max}

X_{min} : Lower limit of the variable

X_{max} : Upper limit of the variable

The intermediate values were coded as –1, 0, and 1. The parameters with their limits and notations are given in Table 1. The design matrix chosen to conduct the experiments was a five level, four factor central composite rotatable design consisting of 31 sets of coded conditions and comprising a half replication $2^4 = 16$ factorial design plus 8 star points and 7 centre pointsas given in Table 2. All parameters at the intermediate level (0) constitute the centre points while the combination of each parameter at either its lower level (–2) or its higher level (+2) with the other two parameters at the intermediate level constitute the star points. Thus the 31 experimental runs allow the estimation of linear, quadratic, and two way interactive effects of the parameters on odometry error (Montgomery, 2000).

GENETIC ALGORITHM

Genetic Algorithm (GA) is an optimization techniquein which a natural evolution process is simulated based on the principles of natural genetics and Darwin´s theory of survival of the fittest. The genetic operators such as reproduction, crossover and mutation are used in the genetic search procedure. The initial step in GA is to define new populations from existing populations. The populations are then ranked according to the fitness function (objective function) value. Through the selection procedure, the best individuals (parents) are preferred for reproduction. Children are then produced either by making random changes to a single parent (mutation) or by joining the pair of parents (crossover). The current population is replaced with the children to make the next generation. At each generation, GA executes a series of computations on the current population to produce a new one. Genetic algorithm runs until an appropriate solution has been obtained.

RESULTS AND DISCUSSION

Development of Mathematical Model

The general form of a quadratic polynomial which gives the relation between response surface ‘*Y*’ and theprocess variable ‘*X*’ is given in Equation (4).

$$Y = a_0 + \sum_{i=1}^{4} a_i X_i + \sum_{i=1}^{4} a_{ii} X_i^2 + \sum_{i<j}^{4} a_{ij} X_i X_j \quad ...(4)$$

Table 1: Parameters and Their Levels

Parameters and Notations	Units	Levels				
		–2	–1	0	1	2
Payload (L)	kg	0.2	0.4	0.6	0.8	1
Speed (S)	mm/sec	100	120	140	160	180
Diameter of Wheel (D)	mm	40	50	60	70	80
Thickness of Wheel (T)	mm	5	10	15	20	25

Table 2: Experimental Design-Central Composite Rotatable Design Matrix

Exp. No.	Control Parameters				Odometry Error (rad)		% Error
	L	S	D	T	Measured	Predicted	
1.	–1	–1	–1	–1	0.03565	0.03590	–0.69
2.	1	–1	–1	–1	0.01785	0.01756	1.64
3.	–1	1	–1	–1	0.03425	0.03399	0.75
4.	1	1	–1	–1	0.01485	0.01465	1.34
5.	–1	–1	1	–1	0.03205	0.03106	3.08
6.	1	–1	1	–1	0.01765	0.01722	2.43
7.	–1	1	1	–1	0.03425	0.03405	0.59
8.	1	1	1	–1	0.01895	0.01921	–1.35
9.	–1	–1	–1	1	0.03235	0.03174	1.89
10.	1	–1	–1	1	0.02185	0.02215	–1.36
11.	–1	1	–1	1	0.02955	0.03009	–1.81
12.	1	1	–1	1	0.01885	0.01949	–3.41
13.	–1	–1	1	1	0.02625	0.02655	–1.15
14.	1	–1	1	1	0.02155	0.02146	0.42
15.	–1	1	1	1	0.02985	0.02979	0.21
16.	1	1	1	1	0.02385	0.02370	0.64
17.	–2	0	0	0	0.03755	0.03794	–1.04
18.	2	0	0	0	0.01365	0.01351	1.03
19.	0	–2	0	0	0.02485	0.02551	–2.64
20.	0	2	0	0	0.02625	0.02584	1.57
21.	0	0	–2	0	0.03025	0.02994	1.02
22.	0	0	2	0	0.02875	0.02931	–1.94
23.	0	0	0	–2	0.02015	0.02096	–4.01
24.	0	0	0	2	0.02185	0.02129	2.56
25.	0	0	0	0	0.02085	0.02092	–0.34
26.	0	0	0	0	0.02125	0.02092	1.55
27.	0	0	0	0	0.02065	0.02092	–1.31
28.	0	0	0	0	0.02185	0.02092	4.25
29.	0	0	0	0	0.02015	0.02092	–3.83
30.	0	0	0	0	0.02085	0.02092	–0.34
31.	0	0	0	0	0.02085	0.02092	–0.34

where a_0 = constant, a_i = linear term coefficient, a_{ii} = quadratic term coefficient and a_{ij} = interaction term coefficient. The values of the coefficients of the polynomials were calculated using the multiple regression method. A statistical analysis software MINITAB was used to calculate the values ofthese coefficients. The second order mathematical model was developed for Odometry error (Oe) as given in Equation (5).

$$Oe = 0.020921 - 0.006108L + 0.000083S - 0.0000158D + 0.000083T + 0.001201L^2 + 0.001188S^2 + 0.002176D^2 + 0.000051T^2 - 0.00025LS + 0.001125LD + 0.002187LT + 0.0012225SD + 0.000063ST - 0.000088DT \quad ...(5)$$

where

L : Payload in kg

S : Speed of robot in mm/sec

D : Diameter of wheel in mm

T : Thickness of wheel in mm

Adequacy of the Model

The adequacy of the model is tested using the analysis of variance (ANOVA). As per the ANOVA technique (Box and Hunter, 1978), the model can be considered to be adequate if the calculated value of F-ratio of the model should not exceed the standard tabulated value of F-ratio for a desired level of confidence (95%). From the values in Table 3, it can be deduced that the current model is adequate. It is evident from the Table 2 that the error between the experimental value and predicted value is less than 5%.

Validation of the Model

The validation of the model is checked for certain levels of the parameters, which have not been included inthe experimental design. The validations of the experimental data are shown in Table 4. From this table, it is observed that the error between the measured value and predicted value is less than 5%, which confirms the validity of the model.

Effects of Parameters on Odometry Error

The direct effects of various parameters like payload, speed, diameter of wheel and thickness of wheel on odometry error were analyzed by conducting experiments and the results are shown in graphs. The influence of

Table 3: Adequacy of the Model

Response	DOF		F-Ratio		Remarks
	Lack of Fit	Error Term	Model	Standard	
Odometry Error	10	6	1.8	4.06	Model is Adequate

Table 4: Result of Conformity Tests for Validating the Model

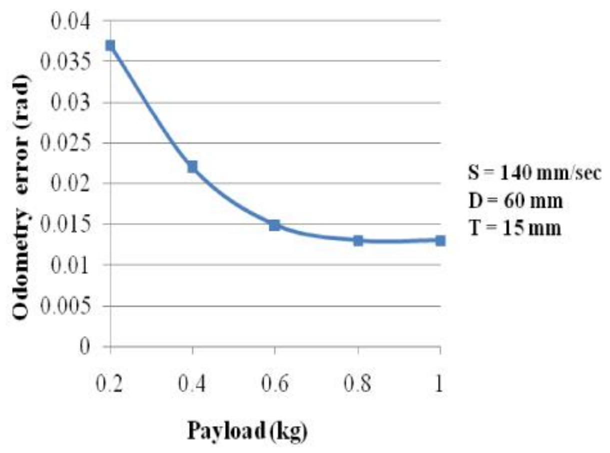
Exp. No.	Control Factors				Odometry Error (rad)		% Error
	L	S	D	T	Measured	Predicted	
1.	–2	2	–2	2	0.04385	0.04472	–1.97
2.	2	–2	2	–2	0.01955	0.01899	2.88

particular parameter in different levels over odometry error was analyzed in the following sections by considering other parameters at middle level.

Effect of Payload

From Figure 3, it was observed that the odometry error decreased with the increase of payload up to 0.8 kg. Also, it was observed that there was no significant change in the odometry error beyond it. So, the odometry error could be lesser between 0.8 kg and 1 kg. The increase in payload considerably reduces the wheel slippage and tottering of robot that leads to less odometry error.

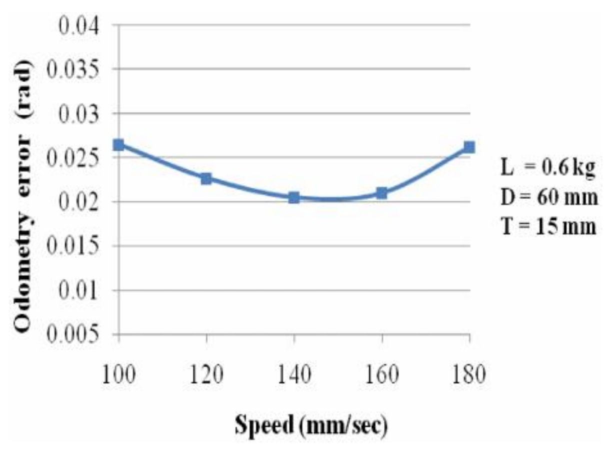
Figure 3: Payload on Odometry Error



Effect of Speed

From Figure 4, it was noted that when the speed increased from 100 mm/sec, the odometry error decreased significantly and started increasing after certain level. So, it is clearly understood that the error is minimum between 140 mm/sec and 160 mm/sec. When the robot is moving with slower speed in practical conditions, the possibilities of wheel distortions, vibrations and wheel slippage are appreciably less which causes very minimal odometry error.

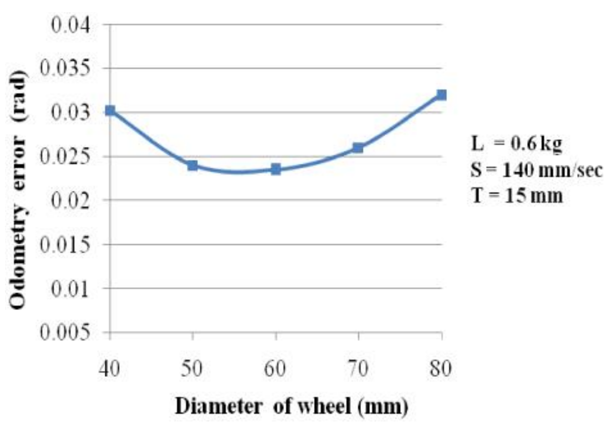
Figure 4: Speed on Odometry Error



Effect of Diameter of Wheel

From Figure 5, it was noticed that the odometry error decreased when the diameter of wheel increased from 40 mm to 50 mm and increased when diameter increased further. It is clear that the odometry error is lesser between 50 to 60 mm of wheel diameter. The diameter of wheel should not be very high as well as very low for the reduction of odometry error. The lesser and larger diameters of wheel lead to the possibilities for wheel turn at maximum speeds and jerks at slower speeds respectively which cause deviation from linear path.

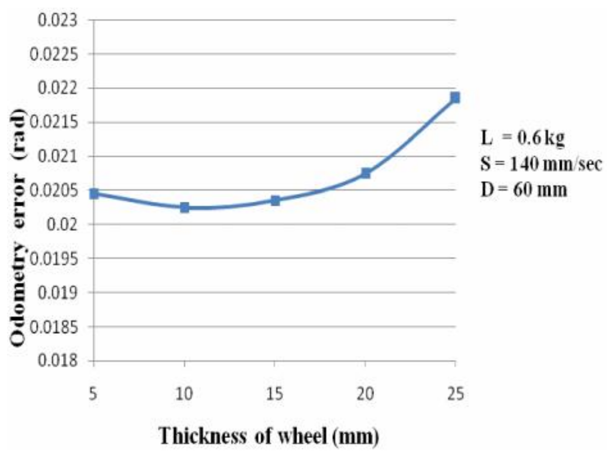
Figure 5: Diameter of Wheel on Odometry Error



Effect of Thickness of Wheel

From Figure 6, it was observed that there wasa decrease in odometry error when the thickness varied from 5 mm to 10 mm and further increased with the increase in thickness. So, it is evident that the minimum odometry error seems to be in the range 10-15 mm of thickness. The lesser contact area of wheel on the floor due to smaller thickness provides the accurate wheel base for the odometry calculation that leads to minimal odometry error.

Figure 6: Thickness of Wheel on Odometry Error



Optimum Condition Through GA

From the effects of parameters on odometry error, the optimum ranges of all parameters were found for minimum odometry error. In order to obtain optimum level of each parameter for minimal odometry error, MATLAB genetic algorithm tool was used in this study. The mathematical model givenin Equation (5) was used as the fitness function (objective function). The bound (constraint)for all parameters were fixed as follows.

Payload, L (kg) : $0.2 \leq L \leq 1$

Speed, S (mm/s) : $100 \leq S \leq 180$

Diameter of wheel, D (mm) : $40 \leq D \leq 80$

Thickness of wheel, T (mm) : $5 \leq T \leq 25$

Genetic algorithm was run for the evolutionary parameters such as population type (double vector), population size (20), fitness selection function (stochastic), probability of crossover (0.8) and probability of mutation (0.03). It was noted that the fitness value decreased through generations as shown in Figure 7and an optimized odometry error (0.010254 rad) was obtained in the final (70th) generation. The optimum condition in the final generation was noted as follows.

Payload : 1 kg

Speed : 143.5 mm/sec

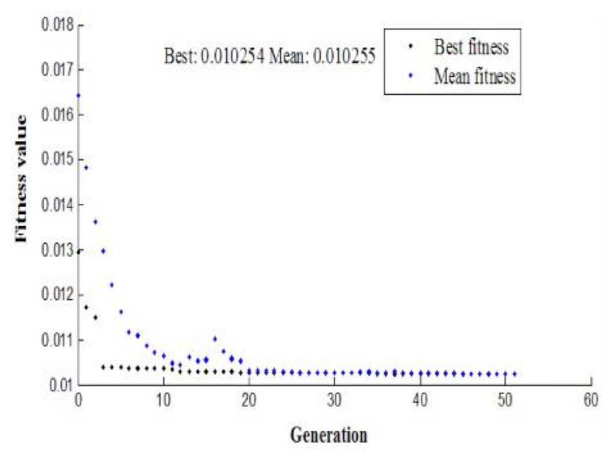
Diameter of wheel : 55.19 mm

Thickness of wheel : 10.97 mm

Confirmatory Test for GA Optimum Condition

A confirmatory experiment was conducted for the optimum condition (L = 1 kg; S = 144 mm/sec; D = 55 mm and T = 11 mm) and odometry error (0.01035 rad) was obtained, which is very close to the GA result.

Figure 7: Fitness Function Values Thorough Generations



CONCLUSION

In this work, experiments were conducted based on central composite rotatable design matrix. A mathematical model was developed for the two wheeled differential drive robot using Response Surface Methodology. Genetic algorithmtool was employed to determine the optimum condition for the better relative positioning, i.e., minimum odometry error. The optimum level of parameters was found as follows.

Payload, L (kg) : 1 kg

Speed, S (mm/s) : 144 mm/sec

Diameter of wheel, D (mm) : 55 mm

Thickness of wheel, T (mm) : 11 mm

The optimum condition was checked through the confirmation experiment. From this study, this optimum parametric setting is suggested for achieving optimum relative positioning.

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APPENDIX

Nomenclature		
D_L	:	Linear distance travelled by left wheel
D_R	:	Linear distance travelled by right wheel
DOF	:	Degree of freedom